Package ‘invLT’

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Type Package

Title Inversion of Laplace-Transformed Functions

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Description Provides two functions for the numerical inversion of Laplace-Transformed functions, returning the value of the standard (time) domain function at a specified value. The first algorithm is the first optimum contour algorithm described by Evans and Chung (2000)[1]. The second algorithm uses the Bromwich contour as per the definition of the inverse Laplace Transform. The latter is unstable for numerical inversion and mainly included for comparison or interest. There are also some additional functions provided for utility, including plotting and some simple Laplace Transform examples, for which there are known analytical solutions. Polar-cartesian conversion functions are included in this package and are used by the inversion functions.
[1] Evans & Chung, 2000: Laplace transform inversions using optimal contours in the complex plane; International Journal of Computer Mathematics v73 pp531-543.

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R topics documented:

BrC.r *Bromwich Contour*

Description

The Bromwich contour with polar co-ordinates (r as a function of ϕ)

Usage

```
BrC.r(phi, gamma = 1)

BrC.ds_dphi(phi, gamma = 1)
```

Arguments

phi	ϕ value
gamma	the Bromwich contour is a straight line and intersects the real axis at γ

Functions

- BrC.r: r
- BrC.ds_dphi: $ds/d\phi$

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iv.opC *Inverse Laplace Transform*

Description

Functionals that numerically invert a Laplace Transform.

Usage

```
iv.opC(L.FUN, t, nterms = 31L, m = 1, fail.val = NA)

iv.opChalf(L.FUN, t, nterms = 16L, m = 1, fail.val = NA)

iv.BrC(L.FUN, t, nterms = 1000L, gamma = 1)
```

Arguments

L.FUN	the Laplace-Transformed function
t	standard (time) domain function at which to evaluate
nterms	number of terms to use in the numerical inversion (odd number safest for iv.opC, even for iv.opChalf)
m	see opC.r documentation
fail.val	value to return in event of failure to converge
gamma	the Bromwich contour is a straight line and intersects the real axis at γ

Details

Optimum contour based on:

Evans & Chung, 2000: Laplace transform inversions using optimal contours in the complex plane *International Journal of Computer Mathematics* v73 pp531-543.

Functions

- iv.opC: inversion using the full optimum contour
- iv.opChalf: for functions which are symmetric about the real axis, it is sufficient to use half the optimum contour and half the number of subdivisions (nterms)
- iv.BrC: inversion using the Bromwich contour (the definition, but very unstable for numerical evaluation - not recommended)

Examples

```
tvals <- seq(-pi/2, pi/2, length.out = 7)
sinvals <- vapply(tvals, iv.opC, complex(1), L.FUN = L.sin)
plot(tvals, Re(sinvals), type = "l")
```

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ivLT.plot *Plot Laplace Transform inversion*

Description

Plots the results of a Laplace Transform inversion at multiple time values.

Usage

```
ivLT.plot(L.FUN, METHOD = iv.opC, tPts = seq(-2, 5, 0.1), nterms = 100,
  ...)
```

Arguments

`L.FUN`	the Laplace-Transformed function
`METHOD`	inversion algorithm to use (iv.opC, iv.opChalf or iv.BrC)
`tPts`	time points at which to plot
`nterms`	number of terms to use in the numerical inversion (odd number safest for `iv.opC`, even for `iv.opChalf`)
`...`	graphical parameters for `plot`

Details

This function is useful for investigating the performance of a Laplace Transform inversion over a range of time values. Use for example with the LT functions provided in with this package (invLT).

Examples

```
ivLT.plot(L.tsq, iv.opC, nterms = 31L)
ivLT.plot(L.tsq, iv.opC, nterms = 1000L)
ivLT.plot(L.tsq, iv.opChalf, nterms = 16L)
ivLT.plot(L.tsq, iv.opChalf, nterms = 1000L)
ivLT.plot(L.tsq, iv.BrC, nterms = 31L)
ivLT.plot(L.tsq, iv.BrC, nterms = 1000L)
```

`L.t` *Laplace Transforms*

Description

Laplace Transforms of common functions. Useful for testing out LT inversion functions and whether sufficient precision is being used.

Usage

```
L.t(p)

L.tsq(p)

L.exp(p)

L.cos(p)

L.sin(p)

L.H(p)
```

Arguments

`p`	Laplace domain variable (commonly called s elsewhere)

Functions

- `L.t`: LT of t
- `L.tsq`: LT of t^2
- `L.exp`: LT of $e^(-t)$
- `L.cos`: LT of $\cos(t)$
- `L.sin`: LT of $\sin(t)$
- `L.H`: LT of Heaviside unit function stepping at 1: (if $p < 1$ 0 else 1)

`opC.r` *Optimum Contour*

Description

The optimum contour with polar co-ordinates (r as a function of ϕ) and complex length increment with ϕ (Evans and Chung, 2000)

Usage

```
opC.r(phi, m = 1, t = 5)

opC.ds_dphi(phi, m = 1, t = 5)
```

Arguments

`phi`	ϕ value
`m`	width of the contour - too small and get too close to singularities on negative x-axis, too large and encounter instability for large positive x
`t`	standard (time) domain variable, also affects contour width

Details

if `t` is set as zero, it is changed to 5 (avoids dividing by 0)

Evans & Chung, 2000: Laplace transform inversions using optimal contours in the complex plane *International Journal of Computer Mathematics* v73 pp531-543.

Functions

- `opC.r`: r
- `opC.ds_dphi`: $ds/d\phi$

`r.xy` *Cartesian to Polar*

Description

Conversion from cartesian to polar co-ordinates

Usage

```
r.xy(x, y)

phi.xy(x, y)
```

Arguments

`x`	x co-ordinate
`y`	y co-ordinate

Value

r or phi respectively from x and y

Functions

- `r.xy`: Returns polar co-ordinate r from cartesian co-ordinates x and y.
- `phi.xy`: Returns polar co-ordinate phi (anti-clockwise rotation from positive x-axis) from cartesian co-ordinates x and y.

`x.rphi` *Polar to Cartesian*

Description

Conversion from polar to cartesian co-ordinates

Usage

```
x.rphi(r, phi)

y.rphi(r, phi)
```

Arguments

`r`	distance from origin
`phi`	anti-clockwise rotation from positive x-axis

Value

x or t respectively from r and phi

Functions

- `x.rphi`: Returns cartesian co-ordinate x from polar co-ordinates r and phi.
- `y.rphi`: Returns cartesian co-ordinate x from polar co-ordinates r and phi.

Index