

# Validation of choplump R package

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## Summary

This document outlines several ways we have tested the choplump R package.

- In Section 1 we calculate the exact choplump tests in two ways. One method is a crude and slower method that is easier to program, and the other method is the faster method which is used for the final `choplump` function. We test both methods with small sample sizes and get the same answers.
- In Section 2 we show that when we use the methods outlined in the `computation` vignette to calculate the usual Wilcoxon Rank sum test (but with many tied zeros), we obtain the same answer as from the previously developed `coin` package (Hothorn, Hornik, van de Wiel and Zeileis, 2006).
- In Section 3 we show that the asymptotic approximation to the p-value closely matches the exact p-value for sample sizes as small as 10 total non-zero responses in both groups.

## 1 Calculate Exact Test Two Ways

In this section, we introduce the `choplumpGeneral` function which is a slow version of the `choplump` test. It's structure closely matches the general description of the choplump test. The key to this function is the `chooseMatrix` function. The call `chooseMatrix(N,M)` produces a `choose(N,M)` by  $N$  matrix with each row a different permutation of  $M$  ones and  $N - M$  zeros.

For example:

```
> chooseMatrix(5,2)
      [,1] [,2] [,3] [,4] [,5]
[1,]    1    1    0    0    0
[2,]    1    0    1    0    0
[3,]    1    0    0    1    0
[4,]    1    0    0    0    1
[5,]    0    1    1    0    0
[6,]    0    1    0    1    0
[7,]    0    1    0    0    1
[8,]    0    0    1    1    0
[9,]    0    0    1    0    1
[10,]   0    0    0    1    1
```

Here is the general chopping function and the general choplump function:

```
> chopGeneral
function (d, W = "W", Z = "Z")
{
  ord <- order(d[, W], d[, Z])
  w <- d[ord, W]
  z <- d[ord, Z]
  M <- length(w[w != 0])
}
```

```

m0 <- length(z[w != 0 & z == 0])
m1 <- M - m0
n1 <- length(z[z == 1])
N <- length(w)
n0 <- N - n1
k0 <- n0 - m0
k1 <- n1 - m1
if (m0/n0 >= m1/n1) {
  a <- 0
  b <- k1 - floor((n1 * k0)/n0)
}
else {
  a <- k0 - floor((n0 * k1)/n1)
  b <- 0
}
wout <- w[(N + 1 - (M + a + b)):N]
zout <- c(rep(0, a), rep(1, b), z[(N + 1 - M):N])
dout <- data.frame(W = wout, Z = zout)
dout
}
<bytecode: 0x0000021b2ffb3340>
<environment: namespace:choplump>

> choplumpGeneral

function (W, Z, testfunc = testfunc.wilcox.ties.general)
{
  w <- W
  z <- Z
  M <- length(w[w != 0])
  m0 <- length(z[w != 0 & z == 0])
  m1 <- M - m0
  n1 <- length(z[z == 1])
  N <- length(w)
  n0 <- N - n1
  k0 <- n0 - m0
  k1 <- n1 - m1
  d <- data.frame(W = w, Z = z)
  T0 <- testfunc(chopGeneral(d))
  cm <- chooseMatrix(N, n1)
  Nperm <- dim(cm)[1]
  Ti <- rep(NA, Nperm)
  for (i in 1:Nperm) {
    d$Z <- cm[i, ]
    Ti[i] <- testfunc(chopGeneral(d))
  }
  p.lower <- length(Ti[Ti <= T0])
  p.upper <- length(Ti[Ti >= T0])
  out <- c(p.lower = p.lower/Nperm, p.upper = p.upper/Nperm,
    p.2sided = min(1, 2 * min(p.lower, p.upper)/Nperm))
}

```

```

    out
  }
<bytecode: 0x0000021b2ffdf9a0>
<environment: namespace:choplump>

```

Now we calculate some p-values from some small data sets using both the `choplumpGeneral` function and the final `choplump` function. We show that both give the same answers.

```

> make.data<-function(N,M,SEED){
+   set.seed(SEED)
+   Z<-rep(0,N)
+   Z[sample(1:N,N/2,replace=FALSE)]<-1
+   test<-data.frame(W=c(rep(0,N-M),abs(rnorm(M))),Z=Z)
+   return(test)
+ }
> test<-make.data(10,6,SEED[1])
> test

      W Z
1 0.00000000 1
2 0.00000000 1
3 0.00000000 0
4 0.00000000 1
5 1.272429321 0
6 0.414641434 0
7 1.539950042 1
8 0.928567035 0
9 0.294720447 1
10 0.005767173 0

> choplumpGeneral(test$W,test$Z,testfunc=testfunc.wilcox.ties.general)

  p.lower  p.upper  p.2sided
0.7380952 0.2936508 0.5873016

> cout<-choplump(W~Z,data=test,use.ranks=TRUE,exact=TRUE)

calculating exact test...
requires 62 evaluations of test statistic

> cout

      Exact Choplump Rank Test

data:  W by Z
p-value = 0.5873
alternative hypothesis: two.sided

> cout$p.values

  p.lower  p.upper  p.2sided
0.7380952 0.2936508 0.5873016

```

Now we show the equivalence of the two functions for the difference in means test. Note to match directions we use the negative of the TDiM function.

```
> testfunc.DiM.general<-function(d){
+   -TDiM(d$W,d$Z)
+ }
> choplumpGeneral(test$W,test$Z,testfunc=testfunc.DiM.general)

  p.lower  p.upper  p.2sided
0.6349206 0.3809524 0.7619048

> choplump(W~Z,data=test,use.ranks=FALSE,exact=TRUE)$p.values

calculating exact test...
requires 62 evaluations of test statistic
  p.lower  p.upper  p.2sided
0.6349206 0.3809524 0.7619048
```

## 2 Calculate Exact Wilcoxon Rank sum test using Similar methods

In this section we show how we can use similar methods to those used in the `choplump` package to calculate either exact or approximate Wilcoxon rank sum test p-values. Then we can check out that the exact p-values match those output from `wilcox_test` in the `coin` package. In the process, we show how our algorithm can be faster in some cases when there are very many zeros.

```
> library(coin)
> test<-make.data(20,12,SEED[1])
> test
```

```
      W Z
1 0.0000000 1
2 0.0000000 1
3 0.0000000 1
4 0.0000000 1
5 0.0000000 0
6 0.0000000 0
7 0.0000000 1
8 0.0000000 0
9 0.48742905 0
10 0.73832471 0
11 0.57578135 1
12 0.30538839 0
13 1.51178117 1
14 0.38984324 1
15 0.62124058 0
16 2.21469989 0
17 1.12493092 1
18 0.04493361 0
19 0.01619026 1
20 0.94383621 0
```

```

> wilcox.manyzeros.exact(W=test$W,Z=test$Z)

  p.lower  p.upper  p.2sided
0.7977224 0.2171296 0.4342592

> test2<-data.frame(W=test$W,Z=as.factor(test$Z))
> wilcox_test(W~Z,data=test2,distribution="exact",alternative="less")

      Exact Wilcoxon-Mann-Whitney Test

data:  W by Z (0, 1)
Z = 0.82004, p-value = 0.7977
alternative hypothesis: true mu is less than 0

> wilcox_test(W~Z,data=test2,distribution="exact",alternative="greater")

      Exact Wilcoxon-Mann-Whitney Test

data:  W by Z (0, 1)
Z = 0.82004, p-value = 0.2171
alternative hypothesis: true mu is greater than 0

> wilcox_test(W~Z,data=test2,distribution="exact",alternative="two.sided")

      Exact Wilcoxon-Mann-Whitney Test

data:  W by Z (0, 1)
Z = 0.82004, p-value = 0.4343
alternative hypothesis: true mu is not equal to 0

> test<-make.data(1000,12,SEED[2])
> t0<-proc.time()
> wilcox.manyzeros.exact(W=test$W,Z=test$Z)

  p.lower  p.upper  p.2sided
0.5341409 0.4793681 0.9587362

> ## time for our algorithm
> proc.time()-t0

  user  system elapsed
  0.03   0.00   0.03

> test2<-data.frame(W=test$W,Z=as.factor(test$Z))
> t1<-proc.time()
> wilcox_test(W~Z,data=test2,distribution="exact",alternative="two.sided")

      Exact Wilcoxon-Mann-Whitney Test

data:  W by Z (0, 1)
Z = 0.0023222, p-value = 0.9587
alternative hypothesis: true mu is not equal to 0

> ## time for coin algorithm
> proc.time()-t1

  user  system elapsed
  0.29   0.03   0.33

```

### 3 Compare Asymptotic Approximation to Exact Calculation

To see how well the approximation performs, we simulate 200 data sets with  $N = 100$  and  $M = 10$ . For this small sample size we can calculate the exact p-values. We randomly assign  $N/2$  of the  $Z_i$  values to 1 and the others are 0. We take pseudo-random numbers for the  $X_i$ , where  $X_i = |X_i^\dagger|$  and  $X_i^\dagger \sim N(0, 1)$ . We plot the bias (approximate p-value minus exact p-value) by the exact p-values in Figure 1, together with the 95% interquantile ranges of the bias. We see that even when  $M$  is as small as 10, the approximation does fairly well, with the 95% interquantile range of the bias for the rank tests equal to  $(-0.0179, 0.003)$ , and the similar statistic for the difference in means tests equal to  $(-0.0192, 0.0197)$ .

### References

Torsten Hothorn, Kurt Hornik, Mark A. van de Wiel and Achim Zeileis (2006). A Lego System for Conditional Inference. *The American Statistician* **60**: 257-263.

Figure 1: Comparison of Asymptotic Approximation and Exact P-values, Circles (solid lines) are Rank Tests and Triangles (dotted lines) are Difference in Means Tests. Lines enclose middle 95% of bias (asymptotic-exact).

