

# Package ‘Rmpfr’

June 3, 2022

**Title** R MPFR - Multiple Precision Floating-Point Reliable

**Version** 0.8-9

**Date** 2022-06-02

**DateNote** Previous CRAN version 0.8-8 on 2022-06-01

**Type** Package

**Description** Arithmetic (via S4 classes and methods) for arbitrary precision floating point numbers, including transcendental (“special”) functions. To this end, the package interfaces to the ‘LGPL’ licensed ‘MPFR’ (Multiple Precision Floating-Point Reliable) Library which itself is based on the ‘GMP’ (GNU Multiple Precision) Library.

**SystemRequirements** gmp ( $\geq 4.2.3$ ), mpfr ( $\geq 3.0.0$ ), pdfcrop (part of TexLive) is required to rebuild the vignettes.

**SystemRequirementsNote** ‘MPFR’ (MP Floating-Point Reliable Library, <https://www.mpfr.org/>) and ‘GMP’ (GNU Multiple Precision library, <https://gmplib.org/>), see `>> README.md`

**Depends** gmp ( $\geq 0.6-1$ ), R ( $\geq 3.5.0$ )

**Imports** stats, utils, methods

**Suggests** MASS, Bessel, polynom, sfsmisc ( $\geq 1.0-20$ ), Matrix

**SuggestsNote** MASS, polynom, sfsmisc: only for vignette; Matrix: test-tools

**Enhances** dfoptim, pracma

**EnhancesNote** mentioned in Rd xrefs

**URL** <https://rmpfr.r-forge.r-project.org/>

**BugReports** [https://r-forge.r-project.org/tracker/?group\\_id=386](https://r-forge.r-project.org/tracker/?group_id=386)

**License** GPL ( $\geq 2$ )

**Encoding** UTF-8

**NeedsCompilation** yes

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 hjkMpfr())

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**Repository** CRAN

**Date/Publication** 2022-06-03 17:10:22 UTC

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Rmpfr-package *R MPFR - Multiple Precision Floating-Point Reliable*

**Description**

Rmpfr provides S4 classes and methods for arithmetic including transcendental ("special") functions for arbitrary precision floating point numbers, here often called "mpfr - numbers". To this end, it interfaces to the LGPL'ed MPFR (Multiple Precision Floating-Point Reliable) Library which itself is based on the GMP (GNU Multiple Precision) Library.

**Details**

Package: Rmpfr  
 Title: R MPFR - Multiple Precision Floating-Point Reliable  
 Version: 0.8-9  
 Date: 2022-06-02  
 DateNote: Previous CRAN version 0.8-8 on 2022-06-01  
 Type: Package  
 Authors@R: c(person("Martin", "Maechler", role = c("aut", "cre"), email = "maechler@stat.math.ethz.ch", com  
 Description: Arithmetic (via S4 classes and methods) for arbitrary precision floating point numbers, including  
 SystemRequirements: gmp (>= 4.2.3), mpfr (>= 3.0.0), pdfcrop (part of TexLive) is required to rebuild the vignettes.  
 SystemRequirementsNote: 'MPFR' (MP Floating-Point Reliable Library, <https://www.mpfr.org/>) and 'GMP' (GNU Multipl  
 Depends: gmp (>= 0.6-1), R (>= 3.5.0)  
 Imports: stats, utils, methods  
 Suggests: MASS, Bessel, polynom, sfsmisc (>= 1.0-20), Matrix  
 SuggestsNote: MASS, polynom, sfsmisc: only for vignette; Matrix: test-tools  
 Enhances: dfoptim, pracma  
 EnhancesNote: mentioned in Rd xrefs  
 URL: <https://rmpfr.r-forge.r-project.org/>  
 BugReports: [https://r-forge.r-project.org/tracker/?group\\_id=386](https://r-forge.r-project.org/tracker/?group_id=386)  
 License: GPL (>= 2)  
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Index of help topics:

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	precisions
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frexpMpfr	Base-2 Representation and Multiplication of Mpfr Numbers
getPrec	Rmpfr - Utilities for Precision Setting, Printing, etc
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mpfrMatrix-class	Classes "mpfrMatrix" and "mpfrArray"
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outer	Base Functions etc, as an Rmpfr version
pbetaI	Accurate Incomplete Beta / Beta Probabilities For Integer Shapes
pmax	Parallel Maxima and Minima
pnorm	Distribution Functions etc (MPFR)
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sapplyMpfr	Apply a Function over a "mpfr" Vector
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str.mpfr	Compactly Show STRucture of Rmpfr Number Object
sumBinomMpfr	(Alternating) Binomial Sums via Rmpfr

unirootR	One Dimensional Root (Zero) Finding - in pure R
zeta	Special Mathematical Functions (MPFR)

Further information is available in the following vignettes:

Maechler_useR_2011-abstr	useR-2011-abstract (source)
Rmpfr-pkg	Arbitrarily Accurate Computation with R Package Rmpfr (source)
log1mexp-note	Accurately Computing $\log(1 - \exp(\cdot))$ - Assessed by Rmpfr (source)

The following (help pages) index does not really mention that we provide *many* methods for mathematical functions, including [gamma](#), [digamma](#), etc, namely, all of R's (S4) Math group (with the only exception of [trigamma](#)), see the list in the examples. Additionally also [pnorm](#), the "error function", and more, see the list in [zeta](#), and further note the first vignette (below).

**Partial index:**

<a href="#">mpfr</a>	Create "mpfr" Numbers (Objects)
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<a href="#">mpfr-class</a>	Class "mpfr" of Multiple Precision Floating Point Numbers
<a href="#">mpfrMatrix-class</a>	Classes "mpfrMatrix" and "mpfrArray"
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<a href="#">optimizeR</a>	High Precision One-Dimensional Optimization
<a href="#">hjkMpfr</a>	Hooke-Jeeves Derivative-Free Minimization R (working for MPFR)

Further information is available in the following vignettes:

Rmpfr-pkg	Arbitrarily Accurate Computation with R: The 'Rmpfr' package (source, pdf)
log1mexp-note	Accurately Computing $\log(1 - \exp(\cdot))$ - Assessed by Rmpfr (source, pdf)

**Author(s)**

Martin Maechler

**References**

MPFR (MP Floating-Point Reliable Library), <https://www.mpfr.org/>

GMP (GNU Multiple Precision library), <https://gmplib.org/>

and see the vignettes mentioned above.

**See Also**

The R package `gmp` for big integer and rational numbers ([bigrational](#)) on which `Rmpfr` now depends.

**Examples**

```
## Using "mpfr" numbers instead of regular numbers...
n1.25 <- mpfr(5, precBits = 256)/4
n1.25

## and then "everything" just works with the desired chosen precision:high
n1.25 ^ c(1:7, 20, 30) ## fully precise; compare with
print(1.25 ^ 30, digits=19)

exp(n1.25)

## Show all math functions which work with "MPFR" numbers (1 exception: trigamma)
getGroupMembers("Math")

## We provide *many* arithmetic, special function, and other methods:
showMethods(classes = "mpfr")
showMethods(classes = "mpfrArray")
```

---

array\_or\_vector-class *Auxiliary Class "array\_or\_vector"*

---

**Description**

"array\_or\_vector" is the class union of `c("array", "matrix", "vector")` and exists for its use in signatures of method definitions.

**Details**

Using "array\_or\_vector" instead of just "vector" in a signature makes an important difference: E.g., if we had `setMethod(crossprod, c(x="mpfr", y="vector"), function(x,y) CPR(x,y))`, a call `crossprod(x, matrix(1:6, 2, 3))` would extend into a call of `CPR(x, as(y, "vector"))` such that `CPR()`'s second argument would simply be a vector instead of the desired  $2 \times 3$  matrix.

**Objects from the Class**

A virtual Class: No objects may be created from it.

**Examples**

```
showClass("array_or_vector")
```

---

asNumeric-methods	<i>Methods for asNumeric(&lt;mpfr&gt;)</i>
-------------------	--

---

**Description**

Methods for function [asNumeric](#) (in package **gmp**).

**Usage**

```
## S4 method for signature 'mpfrArray'
asNumeric(x)
```

**Arguments**

`x` a “number-like” object, here, a [mpfr](#) or typically [mpfrArray](#)one.

**Value**

an R object of type ([typeof](#)) "numeric", a [matrix](#) or [array](#) if `x` had non-NULL dimension [dim\(\)](#).

**Methods**

`signature(x = "mpfrArray")` this method also dispatches for [mpfrMatrix](#) and returns a numeric array.

`signature(x = "mpfr")` for non-array/matrix, `asNumeric(x)` is basically the same as `as.numeric(x)`.

**Author(s)**

Martin Maechler

**See Also**

our lower level (non-generic) [toNum\(\)](#). Further, [asNumeric](#) (package **gmp**), standard R's [as.numeric\(\)](#).

**Examples**

```
x <- (0:7)/8 # (exact)
X <- mpfr(x, 99)
stopifnot(identical(asNumeric(x), x),
           identical(asNumeric(X), x))

m <- matrix(1:6, 3,2)
(M <- mpfr(m, 99) / 5) ##-> "mpfrMatrix"
asNumeric(M) # numeric matrix
stopifnot(all.equal(asNumeric(M), m/5),
           identical(asNumeric(m), m))# remains matrix
```

---

atomicVector-class      *Virtual Class "atomicVector" of Atomic Vectors*

---

**Description**

The `class` "atomicVector" is a *virtual* class containing all atomic vector classes of base `R`, as also implicitly defined via `is.atomic`.

**Objects from the Class**

A virtual Class: No objects may be created from it.

**Methods**

In the `Matrix` package, the "atomicVector" is used in signatures where typically "old-style" "matrix" objects can be used and can be substituted by simple vectors.

**Extends**

The atomic classes "logical", "integer", "double", "numeric", "complex", "raw" and "character" are extended directly. Note that "numeric" already contains "integer" and "double", but we want all of them to be direct subclasses of "atomicVector".

**Author(s)**

Martin Maechler

**See Also**

`is.atomic`, `integer`, `numeric`, `complex`, etc.

**Examples**

```
showClass("atomicVector")
```



**Description**

Computes the Bernoulli numbers in the desired (binary) precision. The computation happens via the `zeta` function and the formula

$$B_k = -k\zeta(1 - k),$$

and hence the only non-zero odd Bernoulli number is  $B_1 = +1/2$ . (Another tradition defines it, equally sensibly, as  $-1/2$ .)

**Usage**

```
Bernoulli(k, precBits = 128)
```

**Arguments**

`k` non-negative integer vector  
`precBits` the precision in *bits* desired.

**Value**

an `mpfr` class vector of the same length as `k`, with *i*-th component the `k[i]`-th Bernoulli number.

**Author(s)**

Martin Maechler

**References**

[https://en.wikipedia.org/wiki/Bernoulli\\_number](https://en.wikipedia.org/wiki/Bernoulli_number)

**See Also**

`zeta` is used to compute them.

The next version of package `gmp` is to contain `BernoulliQ()`, providing exact Bernoulli numbers as big rationals (class "bigq").

**Examples**

```
Bernoulli(0:10)
plot(as.numeric(Bernoulli(0:15)), type = "h")

curve(-x*zeta(1-x), -.2, 15.03, n=300,
      main = expression(-x %% zeta(1-x)))
legend("top", paste(c("even", "odd "), "Bernoulli numbers"),
      pch=c(1,3), col=2, pt.cex=2, inset=1/64)
```

```

abline(h=0,v=0, lty=3, col="gray")
k <- 0:15; k[1] <- 1e-4
points(k, -k*zeta(1-k), col=2, cex=2, pch=1+2*(k%%2))

## They pretty much explode for larger k :
k2 <- 2*(1:120)
plot(k2, abs(as.numeric(Bernoulli(k2))), log = "y")
title("Bernoulli numbers exponential growth")

Bernoulli(10000)# - 9.0494239636 * 10^27677

```

---

Bessel\_mpfr

*Bessel functions of Integer Order in multiple precisions*


---

## Description

Bessel functions of integer orders, provided via arbitrary precision algorithms from the MPFR library.

Note that the computation can be very slow when  $n$  and  $x$  are large (and of similar magnitude).

## Usage

```

Ai(x)
j0(x)
j1(x)
jn(n, x, rnd.mode = c("N", "D", "U", "Z", "A"))
y0(x)
y1(x)
yn(n, x, rnd.mode = c("N", "D", "U", "Z", "A"))

```

## Arguments

<code>x</code>	a <a href="#">numeric</a> or <a href="#">mpfr</a> vector.
<code>n</code>	non-negative integer (vector).
<code>rnd.mode</code>	a 1-letter string specifying how <i>rounding</i> should happen at C-level conversion to MPFR, see <a href="#">mpfr</a> .

## Value

Computes multiple precision versions of the Bessel functions of *integer* order,  $J_n(x)$  and  $Y_n(x)$ , and—when using MPFR library 3.0.0 or newer—also of the Airy function  $Ai(x)$ . Note that currently  $Ai(x)$  is very slow to compute for large  $x$ .

## See Also

[besselJ](#), and [bessely](#) compute the same bessel functions but for arbitrary *real* order and only precision of a bit more than ten digits.

**Examples**

```

x <- (0:100)/8 # (have exact binary representation)
stopifnot(exprs = {
  all.equal(bessely(x, 0), bY0 <- y0(x))
  all.equal(besselJ(x, 1), bJ1 <- j1(x))
  all.equal(yn(0,x), bY0)
  all.equal(jn(1,x), bJ1)
})

mpfrVersion() # now typically 4.1.0
if(mpfrVersion() >= "3.0.0") { ## Ai() not available previously
  print( aix <- Ai(x) )
  plot(x, aix, log="y", type="l", col=2)
  stopifnot(
    all.equal(Ai(0) , 1/(3^(2/3) * gamma(2/3)))
    , # see https://dlmf.nist.gov/9.2.ii
    all.equal(Ai(100), mpfr("2.6344821520881844895505525695264981561e-291"), tol=1e-37)
  )
  two3rd <- 2/mpfr(3, 144)
  print( all.equal(Ai(0), 1/(3^two3rd * gamma(two3rd)), tol=0) ) # 1.7...e-40
  if(Rmpfr::doExtras()) withAutoprint({ # slowish:
    system.time(ai1k <- Ai(1000)) # 1.4 sec (on 2017 lynne)
    stopifnot(all.equal(print(log10(ai1k)),
      -9157.031193409585185582, tol=2e-16)) # seen 8.8..e-17 | 1.1..e-16
  })
} # ver >= 3.0

```

bind-methods

*"mpfr" '...' - Methods for Functions cbind(), rbind()***Description**

`cbind` and `rbind` methods for signature `...` (see [dotsMethods](#) are provided for class `Mnumber`, i.e., for binding numeric vectors and class `"mpfr"` vectors and matrices (`"mpfrMatrix"`) together.

**Usage**

```

cbind(..., deparse.level = 1)
rbind(..., deparse.level = 1)

```

**Arguments**

`...` matrix-/vector-like R objects to be bound together, see the **base** documentation, [cbind](#).

`deparse.level` integer determining under which circumstances column and row names are built from the actual arguments' 'expression', see [cbind](#).

**Value**

typically a ‘matrix-like’ object, here typically of class `"mpfrMatrix"`.

**Methods**

`...` = **"Mnumber"** is used to `(clr)bind` multiprecision “numbers” (inheriting from class `"mpfr"`) together, maybe combined with simple numeric vectors.

`...` = **"ANY"** reverts to `cbind` and `rbind` from package **base**.

**Author(s)**

Martin Maechler

**See Also**

`cbind2`, `cbind`, Documentation in base R’s **methods** package

**Examples**

```
cbind(1, mpfr(6:3, 70)/7, 3:0)
```

---

chooseMpfr

*Binomial Coefficients and Pochhammer Symbol aka Rising Factorial*

---

**Description**

Compute binomial coefficients, `chooseMpfr(a, n)` being mathematically the same as `choose(a, n)`, but using high precision (MPFR) arithmetic.

`chooseMpfr.all(n)` means the vector `choose(n, 1:n)`, using enough bits for exact computation via MPFR. However, `chooseMpfr.all()` is now **deprecated** in favor of `chooseZ` from package **gmp**, as that is now vectorized.

`pochMpfr()` computes the Pochhammer symbol or “rising factorial”, also called the “Pochhammer function”, “Pochhammer polynomial”, “ascending factorial”, “rising sequential product” or “upper factorial”,

$$x^{(n)} = x(x+1)(x+2)\cdots(x+n-1) = \frac{(x+n-1)!}{(x-1)!} = \frac{\Gamma(x+n)}{\Gamma(x)}.$$

**Usage**

```
chooseMpfr(a, n, rnd.mode = c("N", "D", "U", "Z", "A"))
chooseMpfr.all(n, precBits=NULL, k0=1, alternating=FALSE)
pochMpfr(a, n, rnd.mode = c("N", "D", "U", "Z", "A"))
```

**Arguments**

a	a numeric or <code>mpfr</code> vector.
n	an <code>integer</code> vector; if not of length one, n and a are recycled to the same length.
rnd.mode	a 1-letter string specifying how <i>rounding</i> should happen at C-level conversion to MPFR, see <code>mpfr</code> .
precBits	integer or NULL for increasing the default precision of the result.
k0	integer scalar
alternating	logical, for <code>chooseMpfr.all()</code> , indicating if <i>alternating sign</i> coefficients should be returned, see below.

**Value**

For

`chooseMpfr()`, `pochMpfr()`: an `mpfr` vector of length  $\max(\text{length}(a), \text{length}(n))$ ;

`chooseMpfr.all(n, k0)`: a `mpfr` vector of length  $n - k0 + 1$ , of binomial coefficients  $C_{n,m}$  or, if `alternating` is true,  $(-1)^m \cdot C_{n,m}$  for  $m \in k0:n$ .

**Note**

Currently this works via a (C level) for(`i` in 1:n)-loop which really slow for large n, say  $10^6$ , with computational cost  $O(n^2)$ . In such cases, if you need high precision `choose(a, n)` (or `Pochhammer(a, n)`) for large n, preferably work with the corresponding `factorial(mpfr(...))`, or `gamma(mpfr(...))` terms.

**See Also**

`choose(n, m)` (**base R**) computes the binomial coefficient  $C_{n,m}$  which can also be expressed via Pochhammer symbol as  $C_{n,m} = (n - m + 1)^{(m)} / m!$ .

`chooseZ` from package `gmp`; for now, `factorialMpfr`.

For (alternating) binomial sums, directly use `sumBinomMpfr`, as that is potentially more efficient.

**Examples**

```
pochMpfr(100, 4) == 100*101*102*103 # TRUE
a <- 100:110
pochMpfr(a, 10) # exact (but too high precision)
x <- mpfr(a, 70) # should be enough
(px <- pochMpfr(x, 10)) # the same as above (needing only 70 bits)
stopifnot(pochMpfr(a, 10) == px,
          px[1] == prod(mpfr(100:109, 100))) # used to fail

(c1 <- chooseMpfr(1000:997, 60)) # -> automatic "correct" precision
stopifnot(all.equal(c1, choose(1000:997, 60), tolerance=1e-12))

## --- Experimenting & Checking
n.set <- c(1:10, 20, 50:55, 100:105, 200:203, 300:303, 500:503,
          699:702, 999:1001)
```

```

if(!Rmpfr:::doExtras()) { ## speed up: smaller set
  n. <- n.set[-(1:10)]
  n.set <- c(1:10, n.[ c(TRUE, diff(n.) > 1)])
}
C1 <- C2 <- numeric(length(n.set))
for(i.n in seq_along(n.set)) {
  cat(n <- n.set[i.n],":")
  C1[i.n] <- system.time(c.c <- chooseMpfr.all(n) )[1]
  C2[i.n] <- system.time(c.2 <- chooseMpfr(n, 1:n))[1]
  stopifnot(is.whole(c.c), c.c == c.2,
            if(n > 60) TRUE else all.equal(c.c, choose(n, 1:n), tolerance = 1e-15))
  cat(" [0k]\n")
}
matplot(n.set, cbind(C1,C2), type="b", log="xy",
        xlab = "n", ylab = "system.time(.) [s]")
legend("topleft", c("chooseMpfr.all(n)", "chooseMpfr(n, 1:n)"),
       pch=as.character(1:2), col=1:2, lty=1:2, bty="n")

## Currently, chooseMpfr.all() is faster only for large n (>= 300)
## That would change if we used C-code for the *.all() version

## If you want to measure more:
measureMore <- TRUE
measureMore <- FALSE
if(measureMore) { ## takes ~ 2 minutes (on "lynne", Intel i7-7700T, ~2019)
  n.s <- 2^(5:20)
  r <- lapply(n.s, function(n) {
    N <- ceiling(10000/n)
    cat(sprintf("n=%9g => N=%d: ",n,N))
    ct <- system.time(C <- replicate(N, chooseMpfr(n, n/2)))
    cat("[0k]\n")
    list(C=C, ct=ct/N)
  })
  print(ct.n <- t(sapply(r, `[`, "ct")))
  hasSfS <- requireNamespace("sfsmisc")
  plot(ct.n[, "user.self"] ~ n.s, xlab=quote(n), ylab="system.time(.) [s]",
       main = "CPU Time for chooseMpfr(n, n/2)",
       log = "xy", type = "b", axes = !hasSfS)
  if(hasSfS) for(side in 1:2) sfsmisc::eaxis(side)
  summary(fm <- lm(log(ct.n[, "user.self"]) ~ log(n.s), subset = n.s >= 10^4))
  ## --> slope ~ 2 ==> It's O(n^2)
  nn <- 2^seq(11,21, by=1/16) ; Lcol <- adjustcolor(2, 1/2)
  bet <- coef(fm)
  lines(nn, exp(predict(fm, list(n.s = nn))), col=Lcol, lwd=3)
  text(500000,1, substitute(AA %*% n^EE,
                           list(AA = signif(exp(bet[1]),3),
                                   EE = signif( bet[2], 3))), col=2)
} # measure more

```



formatHex

*Flexibly Format Numbers in Binary, Hex and Decimal Format***Description**

Show numbers in binary, hex and decimal format. The resulting character-like objects can be back-transformed to "mpfr" numbers via `mpfr()`.

**Usage**

```
formatHex(x, precBits = min(getPrec(x)), style = "+", expAlign = TRUE)
```

```
formatBin(x, precBits = min(getPrec(x)), scientific = TRUE,
          left.pad = "_", right.pad = left.pad, style = "+", expAlign = TRUE)
```

```
formatDec(x, precBits = min(getPrec(x)), digits = decdigits,
          nsmall = NULL, scientific = FALSE, style = "+",
          decimalPointAlign = TRUE, ...)
```

**Arguments**

<code>x</code>	a numeric or <code>mpfr</code> R object.
<code>precBits</code>	integer, the number of bits of precision, typically derived from <code>x</code> , see <code>getPrec</code> . Numeric, i.e., double precision numbers have 53 bits. For more detail, see <code>mpfr</code> .
<code>style</code>	a single character, to be used in <code>sprintf</code> 's format ( <code>fmt</code> ), immediately after the " sets a sign in the output, i.e., "+" or "-", where as <code>style = " "</code> may seem more standard.
<code>expAlign</code>	<b>logical</b> indicating if for scientific ("exponential") representations the exponents should be aligned to the same width, i.e., zero-padded to the same number of digits.
<code>scientific</code>	<b>logical</b> indicating that <code>formatBin</code> should display the binary representation in scientific notation ( <code>mpfr(3, 5)</code> is displayed as <code>+0b1.1000p+1</code> ). When <code>FALSE</code> , <code>formatBin</code> will display the binary representation in regular format shifted to align binary points ( <code>mpfr(3, 5)</code> is displayed <code>+0b11.000</code> ).
<code>...</code>	additional optional arguments. <code>formatHex</code> , <code>formatBin</code> : <code>precBits</code> is the only <code>...</code> argument acted on. Other <code>...</code> arguments are ignored. <code>formatDec</code> : <code>precBits</code> is acted on. Any argument accepted by <code>format</code> (except <code>nsmall</code> ) is acted on. Other <code>...</code> arguments are ignored.



left.pad, right.pad	characters (one-character strings) that will be used for left- and right-padding of the formatted string when <code>scientific=FALSE</code> . <i>Do not change these unless for display-only purpose !!</i>
nsmall	only used when <code>scientific</code> is false, then passed to <code>format()</code> . If NULL, the default is computed from the range of the non-zero values of <code>x</code> .
digits	integer; the number of decimal digits displayed is the larger of this argument and the internally generated value that is a function of <code>precBits</code> . This is related to but different than <code>digits</code> in <code>format</code> .
decimalPointAlign	logical indicating if padding should be used to ensure that the resulting strings align on the decimal point (".").

### Details

For the hexadecimal representation, when the precision is not larger than double precision, `sprintf()` is used directly, otherwise `formatMpfr()` is used and post processed. For the binary representation, the hexadecimal value is calculated and then edited by substitution of the binary representation of the hex characters coded in the `HextoBin` vector. For binary with `scientific=FALSE`, the result of the `scientific=TRUE` version is edited to align binary points. For the decimal representation, the hexadecimal value is calculated with the specified precision and then sent to the `format` function for `scientific=FALSE` or to the `sprintf` function for `scientific=TRUE`.

### Value

a character vector (or matrix) like `x`, say `r`, containing the formatted representation of `x`, with a `class` (unless `left.pad` or `right.pad` were not `"_"`). In that case, `formatHex()` and `formatBin()` return class `"Ncharacter"`; for that, `mpfr(.)` has a method and will basically return `x`, i.e., work as *inverse* function.

Since **Rmpfr** version 0.6-2, the S3 class `"Ncharacter"` extends `"character"`. (`class(.)` is now of length two and `class(.)[2]` is `"character"`). There are simple `[]` and `print` methods; modifying or setting `dim` works as well.

### Author(s)

Richard M. Heiberger <rmh@temple.edu>, with minor tweaking by Martin M.

### References

R FAQ 7.31: Why doesn't R think these numbers are equal? `system.file("../..doc/FAQ")`

### See Also

`mpfr`, `sprintf`

**Examples**

```

FourBits <- mpfr(matrix(0:31, 8, 4, dimnames = list(0:7, c(0,8,16,24))),
                 precBits=4) ## 4 significant bits

FourBits

formatHex(FourBits)
formatBin(FourBits, style = " ")
formatBin(FourBits, scientific=FALSE)
formatDec(FourBits)

## as "Ncharacter" 'inherits from' "character", this now works too :
data.frame(Dec = c( formatDec(FourBits) ), formatHex(FourBits),
           Bin = formatBin(FourBits, style = " "))

FBB <- formatBin(FourBits) ; clB <- class(FBB)
(nFBB <- mpfr(FBB))
stopifnot(class(FBB)[1] == "Ncharacter",
          all.equal(nFBB, FourBits, tol=0))

FBH <- formatHex(FourBits) ; clH <- class(FBH)
(nFBH <- mpfr(FBH))
stopifnot(class(FBH)[1] == "Ncharacter",
          all.equal(nFBH, FourBits, tol=0))

## Compare the different "formattings" (details will change, i.e. improve!)% FIXME
M <- mpfr(c(-Inf, -1.25, 1/(-Inf), NA, 0, .5, 1:2, Inf), 3)
data.frame(fH = formatHex(M), f16 = format(M, base=16),
          fB = formatBin(M), f2 = format(M, base= 2),
          fD = formatDec(M), f10 = format(M), # base = 10 is default
          fSci= format(M, scientific=TRUE),
          fFix= format(M, scientific=FALSE))

## Other methods ("[, t()) also work :
stopifnot(dim(F1 <- FBB[, 1, drop=FALSE]) == c(8,1), identical(class( F1), clB),
          dim(t(F1)) == c(1,8), identical(class(t(F1)),clB),
          is.null(dim(F.2 <- FBB[,2])), identical(class( F.2), clB),
          dim(F22 <- FBH[1:2, 3:4]) == c(2,2), identical(class(F22), clH),
          identical(class(FBH[2,3]), clH), is.null(dim(FBH[2,3])),
          identical(FBH[2,3:4], F22[2,]),
          identical(FBH[2,3], unname(FBH[,3][2])),
          TRUE)

TenFrac <- matrix((1:10)/10, dimnames=list(1:10, expression(1/x)))
TenFrac9 <- mpfr(TenFrac, precBits=9) ## 9 significant bits
TenFrac9
formatHex(TenFrac9)
formatBin(TenFrac9)
formatBin(TenFrac9, scientific=FALSE)
formatDec(TenFrac9)
formatDec(TenFrac9, precBits=10)

```

## Description

Flexible formatting of “multiprecision numbers”, i.e., objects of class `mpfr`. `formatMpfr()` is also the `mpfr` method of the generic `format` function.

The `formatN()` methods for `mpfr` numbers renders them differently than their double precision equivalents, by appending “\_M”.

Function `.mpfr2str()` is the low level work horse for `formatMpfr()` and hence all `print()`ing of “`mpfr`” objects.

## Usage

```
formatMpfr(x, digits = NULL, trim = FALSE, scientific = NA,
           maybe.full = (!is.null(digits) && is.na(scientific)) || isFALSE(scientific),
           base = 10, showNeg0 = TRUE, max.digits = Inf,
           big.mark = "", big.interval = 3L,
           small.mark = "", small.interval = 5L,
           decimal.mark = ".",
           exponent.char = if(base <= 14) "e" else if(base <= 36) "E" else "|e",
           exponent.plus = TRUE,
           zero.print = NULL, drop0trailing = FALSE, ...)
```

```
## S3 method for class 'mpfr'
formatN(x, drop0trailing = TRUE, ...)
```

```
.mpfr2str(x, digits = NULL, maybe.full = !is.null(digits), base = 10L)
```

## Arguments

<code>x</code>	an MPFR number (vector or array).
<code>digits</code>	how many significant digits (in the base chosen!) are to be used in the result. The default, <code>NULL</code> , uses enough digits to represent the full precision, often one or two digits more than “you” would expect. For bases 2,4,8,16, or 32, MPFR requires <code>digits</code> at least 2. For such bases, <code>digits = 1</code> is changed into 2, with a message.
<code>trim</code>	logical; if <code>FALSE</code> , numbers are right-justified to a common width: if <code>TRUE</code> the leading blanks for justification are suppressed.
<code>scientific</code>	either a logical specifying whether MPFR numbers should be encoded in scientific format (“exponential representation”), or an integer penalty (see <code>options("scipen")</code> ). Missing values correspond to the current default penalty.
<code>maybe.full</code>	logical, passed to <code>.mpfr2str()</code> .

base	an integer in 2, 3, ..., 62; the base (“basis”) in which the numbers should be represented. Apart from the default base 10, binary (base = 2) or hexadecimal (base = 16) are particularly interesting.
showNeg0	logical indicating if “ <b>negative</b> ” zeros should be shown with a “-”. The default, TRUE is intentionally different from <code>format(&lt;numeric&gt;)</code> .
exponent.char	the “exponent” character to be used in scientific notation. The default takes into account that for base $B \geq 15$ , “e” is part of the (mantissa) digits and the same is true for “E” when $B \geq 37$ .
exponent.plus	<b>logical</b> indicating if “+” should be for positive exponents in exponential (aka “scientific”) representation. This used to be hardcoded to FALSE; the new default is compatible to R’s <code>format()</code> ing of numbers and helps to note visually when exponents are in use.
max.digits	a (large) positive number to limit the number of (mantissa) digits, notably when digits is NULL (as by default). Otherwise, a numeric digits is <i>preferred</i> to setting max.digits (which should not be smaller than digits).
big.mark, big.interval, small.mark, small.interval, decimal.mark, zero.print, drop0trailing	used for prettying decimal sequences, these are passed to <code>prettyNum</code> and that help page explains the details.
...	further arguments passed to or from other methods.

## Value

a character vector or array, say `cx`, of the same length as `x`. Since Rmpfr version 0.5-3 (2013-09), if `x` is an `mpfrArray`, then `cx` is a character `array` with the same `dim` and `dimnames` as `x`.

Note that in scientific notation, the integer exponent is always in *decimal*, i.e., base 10 (even when base is not 10), but of course meaning base powers, e.g., in base 32, “u.giE3” is the same as “ugi0” which is  $32^3$  times “u.gi”. This is in contrast, e.g., with `sprintf("%a", x)` where the powers after “p” are powers of 2.

## Note

Currently, `formatMpfR(x, scientific = FALSE)` does *not work correctly*, e.g., for `x <- Const("pi", 128) * 2^c(-200, 200)`, i.e., it uses the scientific / exponential-style format. This is considered bogus and hopefully will change.

## Author(s)

Martin Maechler

## References

The MPFR manual’s description of ‘`mpfr_get_str()`’ which is the C-internal workhorse for `.mpfr2str()` (on which `formatMpfR()` builds).

**See Also**

`mpfr` for creation and the `mpfr` class description with its many methods. The `format` generic, and the `prettyNum` utility on which `formatMpfr` is based as well. The S3 generic function `formatN` from package `gmp`.

`.mpfr_formatinfo(x)` provides the (cheap) non-string parts of `.mpfr2str(x)`; the (base 2) exp exponents are also available via `.mpfr2exp(x)`.

**Examples**

```
## Printing of MPFR numbers uses formatMpfr() internally.
## Note how each components uses the "necessary" number of digits:
(x3 <- c(Const("pi", 168), mpfr(pi, 140), 3.14) )
format(x3[3], 15)
format(x3[3], 15, drop0 = TRUE)# "3.14" .. dropping the trailing zeros
x3[4] <- 2^30
x3[4] # automatically drops trailing zeros
format(x3[1], dig = 41, small.mark = "") # (41 - 1 = ) 40 digits after "."

rbind(formatN(x3, digits = 15),
       formatN(as.numeric(x3), digits = 15))

(Zero <- mpfr(c(0,1/-Inf), 20)) # 0 and "-0"
xx <- c(Zero, 1:2, Const("pi", 120), -100*pi, -.00987)
format(xx, digits = 2)
format(xx, digits = 1, showNeg0 = FALSE)# "-0" no longer shown

## Output in other bases :
formatMpfr(mpfr(10^6, 40), base=32, drop0trailing=TRUE)
## "ugi0"
mpfr("ugi0", base=32) #-> 1'000'000

## This now works: The large number shows "as" large integer:
x <- Const("pi", 128) * 2^c(-200,200)
formatMpfr(x, scientific = FALSE) # was 1.955...e-60 5.048...e+60

i32 <- mpfr(1:32, precBits = 64)
format(i32, base= 2, drop0trailing=TRUE)
format(i32, base= 16, drop0trailing=TRUE)
format(1/i32, base= 2, drop0trailing=TRUE)# using scientific notation for [17..32]
format(1/i32, base= 32)
format(1/i32, base= 62, drop0trailing=TRUE)
format(mpfr(2, 64)^(1:16), base=16, drop0trailing=TRUE)
```

**Description**

MPFR - versions of the C99 (and POSIX) standard C (and C++) mathlib functions `frexp()` and `ldexp()`.

`frexpMpfr(x)` computes base-2 exponent  $e$  and “mantissa”, or *fraction*  $r$ , such that  $x = r * 2^e$ , where  $r \in [0.5, 1)$  (unless when  $x$  is in `c(0, -Inf, Inf, NaN)` where  $r == x$  and  $e$  is 0), and  $e$  is integer valued.

`ldexpMpfr(f, E)` is the *inverse* of `frexpMpfr()`: Given fraction or mantissa  $f$  and integer exponent  $E$ , it returns  $x = f * 2^E$ . Viewed differently, it's the fastest way to multiply or divide MPFR numbers with  $2^E$ .

**Usage**

```
frexpMpfr(x,   rnd.mode = c("N", "D", "U", "Z", "A"))
ldexpMpfr(f, E, rnd.mode = c("N", "D", "U", "Z", "A"))
```

**Arguments**

<code>x</code>	numeric (coerced to double) vector.
<code>f</code>	numeric fraction (vector), in $[0.5, 1)$ .
<code>E</code>	integer valued, exponent of 2, i.e., typically in $(-1024-50):1024$ , otherwise the result will underflow to 0 or overflow to $\pm \text{Inf}$ .
<code>rnd.mode</code>	a 1-letter string specifying how <i>rounding</i> should happen at C-level conversion to MPFR, see <a href="#">mpfr</a> .

**Value**

`frexpMpfr` returns a [list](#) with named components `r` (of class `mpfr`) and `e` (integer valued, of type integer is small enough, "double" otherwise).

**Author(s)**

Martin Maechler

**References**

On unix-alikes, typically `man frexp` and `man ldexp`

**See Also**

Somewhat related, [.mpfr2exp\(\)](#).

**Examples**

```
set.seed(47)
x <- c(0, 2^(-3:3), (-1:1)/0,
      sort(rlnorm(2^12, 10, 20) * sample(c(-1,1), 512, replace=TRUE)))
head(xM <- mpfr(x, 128), 11)
str(rFM <- frexpMpfr(xM))
```

```

d.fr <- with(rFM, data.frame(x=x, r=asNumeric(r), e=e))
head(d.fr , 16)
tail(d.fr)
ar <- abs(rFM$r)
stopifnot(0.5 <= ar[is.finite(x) & x != 0], ar[is.finite(x)] < 1,
          is.integer(rFM$e))
ldx <- mpfr(with(rFM, ldexpMpfr(r, e)))
(iN <- which(is.na(x))) # 10
stopifnot(exprs = {
  all.equal(xM, ldx, tol = 2^-124) # allow 4 bits loss, but apart from the NA, even:
  identical(xM[-iN], ldx[-iN])
  is.na(xM [iN])
  is.na(ldx[iN])
})

```

gmp-conversions

*Conversion Utilities gmp <-> Rmpfr***Description**

Coerce from and to big integers ([bigz](#)) and [mpfr](#) numbers.

Further, coerce from big rationals ([bigq](#)) to [mpfr](#) numbers.

**Usage**

```

.bigz2mpfr(x, precB = NULL, rnd.mode = c('N','D','U','Z','A'))
.bigq2mpfr(x, precB = NULL, rnd.mode = c('N','D','U','Z','A'))
.mpfr2bigz(x, mod = NA)

```

**Arguments**

<code>x</code>	an R object of class <code>bigz</code> , <code>bigq</code> or <code>mpfr</code> respectively.
<code>precB</code>	precision in bits for the result. The default, <code>NULL</code> , means to use the <i>minimal</i> precision necessary for correct representation.
<code>rnd.mode</code>	a 1-letter string specifying how <i>rounding</i> should happen at C-level conversion to MPFR, see details of <a href="#">mpfr</a> .
<code>mod</code>	a possible modulus, see <a href="#">as.bigz</a> in package <b>gmp</b> .

**Details**

Note that we also provide the natural (S4) coercions, `as(x, "mpfr")` for `x` inheriting from class `"bigz"` or `"bigq"`.

**Value**

a numeric vector of the same length as `x`, of the desired class.

**See Also**

`mpfr()`, `as.bigz` and `as.bigq` in package **gmp**.

**Examples**

```
S <- gmp::Stirling2(50,10)
show(S)
SS <- S * as.bigz(1:3)^128
stopifnot(all.equal(log2(SS[2]) - log2(S), 128, tolerance=1e-15),
          identical(SS, .mpfr2bigz(.bigz2mpfr(SS))))

.bigz2mpfr(S)          # 148 bit precision
.bigz2mpfr(S, precB=256) # 256 bit

## rational --> mpfr:
sq <- SS / as.bigz(2)^100
MP <- as(sq, "mpfr")
stopifnot(identical(MP, .bigq2mpfr(sq)),
          SS == MP * as(2, "mpfr")^100)
```

---

hjkMpfr

*Hooke-Jeeves Derivative-Free Minimization R (working for MPFR)*


---

**Description**

An implementation of the Hooke-Jeeves algorithm for derivative-free optimization.

This is a slight adaption `hjk()` from package **dfoptim**.

**Usage**

```
hjkMpfr(par, fn, control = list(), ...)
```

**Arguments**

<code>par</code>	Starting vector of parameter values. The initial vector may lie on the boundary. If <code>lower[i]=upper[i]</code> for some <code>i</code> , the <code>i</code> -th component of the solution vector will simply be kept fixed.
<code>fn</code>	Nonlinear objective function that is to be optimized. A scalar function that takes a real vector as argument and returns a scalar that is the value of the function at that point.
<code>control</code>	<code>list</code> of control parameters. See <b>Details</b> for more information.
<code>...</code>	Additional arguments passed to <code>fn</code> .



## Details

Argument `control` is a list specifying changes to default values of algorithm control parameters. Note that parameter names may be abbreviated as long as they are unique.

The list items are as follows:

`tol` Convergence tolerance. Iteration is terminated when the step length of the main loop becomes smaller than `tol`. This does *not* imply that the optimum is found with the same accuracy. Default is `1.e-06`.

`maxfeval` Maximum number of objective function evaluations allowed. Default is `Inf`, that is no restriction at all.

`maximize` A logical indicating whether the objective function is to be maximized (`TRUE`) or minimized (`FALSE`). Default is `FALSE`.

`target` A real number restricting the absolute function value. The procedure stops if this value is exceeded. Default is `Inf`, that is no restriction.

`info` A logical variable indicating whether the step number, number of function calls, best function value, and the first component of the solution vector will be printed to the console. Default is `FALSE`.

If the minimization process threatens to go into an infinite loop, set either `maxfeval` or `target`.

## Value

A `list` with the following components:

<code>par</code>	Best estimate of the parameter vector found by the algorithm.
<code>value</code>	value of the objective function at termination.
<code>convergence</code>	indicates convergence ( <code>TRUE</code> ) or not ( <code>FALSE</code> ).
<code>feval</code>	number of times the objective fn was evaluated.
<code>niter</code>	number of iterations (“steps”) in the main loop.

## Note

This algorithm is based on the Matlab code of Prof. C. T. Kelley, given in his book “Iterative methods for optimization”. It has been implemented for package **dfoptim** with the permission of Prof. Kelley.

This version does not (yet) implement a cache for storing function values that have already been computed as searching the cache makes it slower.

## Author(s)

Hans W Borchers <hwborchers@googlemail.com>; for **Rmpfr**: John Nash, June 2012. Modifications by Martin Maechler.

## References

- C.T. Kelley (1999), Iterative Methods for Optimization, SIAM.  
 Quarteroni, Sacco, and Saleri (2007), Numerical Mathematics, Springer.

**See Also**

Standard R's `optim`; `optimizeR` provides *one*-dimensional minimization methods that work with `mpfr`-class numbers.

**Examples**

```
## simple smooth example:
ff <- function(x) sum((x - c(2:4))^2)
str(rr <- hjkMpfr(rep(mpfr(0,128), 3), ff, control=list(info=TRUE)))

## Hooke-Jeeves solves high-dim. Rosenbrock function {but slowly!}
rosenbrock <- function(x) {
  n <- length(x)
  sum (100*((x1 <- x[1:(n-1)])^2 - x[2:n])^2 + (x1 - 1)^2)
}
par0 <- rep(0, 10)
str(rb.db <- hjkMpfr(rep(0, 10), rosenbrock, control=list(info=TRUE)))

## rosenbrock() is quite slow with mpfr-numbers:
str(rb.M. <- hjkMpfr(mpfr(numeric(10), prec=128), rosenbrock,
  control = list(tol = 1e-8, info=TRUE)))

## Hooke-Jeeves does not work well on non-smooth functions
nsf <- function(x) {
  f1 <- x[1]^2 + x[2]^2
  f2 <- x[1]^2 + x[2]^2 + 10 * (-4*x[1] - x[2] + 4)
  f3 <- x[1]^2 + x[2]^2 + 10 * (-x[1] - 2*x[2] + 6)
  max(f1, f2, f3)
}
par0 <- c(1, 1) # true min 7.2 at (1.2, 2.4)
h.d <- hjkMpfr(par0, nsf) # fmin=8 at xmin=(2,2)

## and this is not at all better (but slower!)
h.M <- hjkMpfr(mpfr(c(1,1), 128), nsf, control = list(tol = 1e-15))

## --> demo(hjkMpfr) # -> Fletcher's chebyquad function m = n -- residuals
```

---

 igamma

*Incomplete Gamma Function*


---

**Description**

For MPFR version  $\geq 3.2.0$ , the following MPFR library function is provided: `mpfr_gamma_inc(a, x)`, the R interface of which is `igamma(a, x)`, where `igamma(a, x)` is the “upper” incomplete gamma function

$$\Gamma(a, x) := \Gamma(a) - \gamma(a, x),$$

where

$$\gamma(a, x) := \int_0^x t^{a-1} e^{-t} dt,$$

and hence

$$\Gamma(a, x) := \int_x^\infty t^{a-1} e^{-t} dt,$$

and

$$\Gamma(a) := \gamma(a, \infty).$$

As R's `pgamma(x, a)` is

$$\text{pgamma}(x, a) := \gamma(a, x) / \Gamma(a),$$

we get

$$\text{igamma}(a, x) == \text{gamma}(a) * \text{pgamma}(x, a, \text{lower.tail}=\text{FALSE})$$

### Usage

```
igamma(a, x, rnd.mode = c("N", "D", "U", "Z", "A"))
```

### Arguments

`a, x` an object of class `mpfr` or `numeric`.  
`rnd.mode` a 1-letter string specifying how *rounding* should happen at C-level conversion to MPFR, see `mpfr`.

### Value

a numeric vector of “common length”, recycling along `a` and `x`.

### Author(s)

R interface: Martin Maechler

### References

NIST Digital Library of Mathematical Functions, section 8.2. <https://dlmf.nist.gov/8.2.i>  
 Wikipedia (2019). *Incomplete gamma function*; [https://en.wikipedia.org/wiki/Incomplete\\_gamma\\_function](https://en.wikipedia.org/wiki/Incomplete_gamma_function)

### See Also

R's `gamma` (function) and `pgamma` (probability distribution).

**Examples**

```
## show how close pgamma() is :
x <- c(seq(0,20, by=1/4), 21:50, seq(55, 100, by=5))
if(mpfrVersion() >= "3.2.0") { print(
  all.equal(igamma(Const("pi", 80), x),
            pgamma(x, pi, lower.tail=FALSE) * gamma(pi),
            tol=0, formatFUN = function(., ...) format(., digits = 7)) #-> 3.13e-16
)
## and ensure *some* closeness:
stopifnot(exprs = {
  all.equal(igamma(Const("pi", 80), x),
            pgamma(x, pi, lower.tail=FALSE) * gamma(pi),
            tol = 1e-15)
})
} # only if MPFR version >= 3.2.0
```

---

integrateR

*One-Dimensional Numerical Integration - in pure R*


---

**Description**

Numerical integration of one-dimensional functions in pure R, with care so it also works for "mpfr"-numbers.

Currently, only classical Romberg integration of order `ord` is available.

**Usage**

```
integrateR(f, lower, upper, ..., ord = NULL,
           rel.tol = .Machine$double.eps^0.25, abs.tol = rel.tol,
           max.ord = 19, verbose = FALSE)
```

**Arguments**

<code>f</code>	an R function taking a numeric or "mpfr" first argument and returning a numeric (or "mpfr") vector of the same length. Returning a non-finite element will generate an error.
<code>lower, upper</code>	the limits of integration. Currently <i>must</i> be finite. Do use "mpfr"-numbers to get higher than double precision, see the examples.
<code>...</code>	additional arguments to be passed to <code>f</code> .
<code>ord</code>	integer, the order of Romberg integration to be used. If this is NULL, as per default, and either <code>rel.tol</code> or <code>abs.tol</code> are specified, the order is increased until convergence.
<code>rel.tol</code>	relative accuracy requested. The default is 1.2e-4, about 4 digits only, see the Note.
<code>abs.tol</code>	absolute accuracy requested.

max.ord	only used, when neither ord or one of rel.tol, abs.tol are specified: Stop Romberg iterations after the order reaches max.ord; may prevent infinite loops or memory explosion.
verbose	logical or integer, indicating if and how much information should be printed during computation.

### Details

Note that arguments after . . . must be matched exactly.

For convergence, *both* relative and absolute changes must be smaller than rel.tol and abs.tol, respectively.

rel.tol cannot be less than  $\max(50 * \text{Machine}\$double.eps, 0.5e-28)$  if abs.tol  $\leq 0$ .

### Value

A list of class "integrateR" (as from standard R's `integrate()`) with a `print` method and components

value	the final estimate of the integral.
abs.error	estimate of the modulus of the absolute error.
subdivisions	for Romberg, the number of function evaluations.
message	"OK" or a character string giving the error message.
call	the matched call.

### Note

f must accept a vector of inputs and produce a vector of function evaluations at those points. The `Vectorize` function may be helpful to convert f to this form.

If you want to use higher accuracy, you *must* set lower or upper to "mpfr" numbers (and typically lower the relative tolerance, rel.tol), see also the examples.

Note that the default tolerances (rel.tol, abs.tol) are not very accurate, but the same as for `integrate`, which however often returns considerably more accurate results than requested. This is typically *not* the case for `integrateR()`.

### Note

We use practically the same print S3 method as `print.integrate`, provided by R, with a difference when the message component is not "Ok".

### Author(s)

Martin Maechler

### References

Bauer, F.L. (1961) Algorithm 60 – Romberg Integration, *Communications of the ACM* **4**(6), p.255.

**See Also**

R's standard, `integrate`, is much more adaptive, also allowing infinite integration boundaries, and typically considerably faster for a given accuracy.

**Examples**

```
## See more from ?integrate
## this is in the region where integrate() can get problems:
integrateR(dnorm,0,2000)
integrateR(dnorm,0,2000, rel.tol=1e-15)
(Id <- integrateR(dnorm,0,2000, rel.tol=1e-15, verbose=TRUE))
Id$value == 0.5 # exactly

## Demonstrating that 'subdivisions' is correct:
Exp <- function(x) { .N <<- .N+ length(x); exp(x) }
.N <- 0; str(integrateR(Exp, 0,1, rel.tol=1e-10), digits=15); .N

### Using high-precision functions -----

## Polynomials are very nice:
integrateR(function(x) (x-2)^4 - 3*(x-3)^2, 0, 5, verbose=TRUE)
# n= 1, 2^n=      2 | I =          46.04, abs.err =      98.9583
# n= 2, 2^n=      4 | I =          20, abs.err =      26.0417
# n= 3, 2^n=      8 | I =          20, abs.err =  7.10543e-15
## 20 with absolute error < 7.1e-15
## Now, using higher accuracy:
I <- integrateR(function(x) (x-2)^4 - 3*(x-3)^2, 0, mpfr(5,128),
                rel.tol = 1e-20, verbose=TRUE)
I ; I$value ## all fine

## with floats:
integrateR(exp,      0      , 1, rel.tol=1e-15, verbose=TRUE)
## with "mpfr":
(I <- integrateR(exp, mpfr(0,200), 1, rel.tol=1e-25, verbose=TRUE))
(I.true <- exp(mpfr(1, 200)) - 1)
## true absolute error:
stopifnot(print(as.numeric(I.true - I$value)) < 4e-25)

## Want absolute tolerance check only (=> set 'rel.tol' very high, e.g. 1):
(Ia <- integrateR(exp, mpfr(0,200), 1, abs.tol = 1e-6, rel.tol=1, verbose=TRUE))

## Set 'ord' (but no '*.tol') --> Using 'ord'; no convergence checking
(I <- integrateR(exp, mpfr(0,200), 1, ord = 13, verbose=TRUE))
```

**Description**

Check which elements of `x[]` are integer valued aka “whole” numbers, including MPFR numbers (class `mpfr`).

**Usage**

```
## S3 method for class 'mpfr'
is.whole(x)
```

**Arguments**

`x` any R vector, here of class `mpfr`.

**Value**

logical vector of the same length as `x`, indicating where `x[.]` is integer valued.

**Author(s)**

Martin Maechler

**See Also**

`is.integer(x)` (**base** package) checks for the *internal* mode or class, not if `x[i]` are integer valued. The `is.whole()` methods in package **gmp**.

**Examples**

```
is.integer(3) # FALSE, it's internally a double
is.whole(3)   # TRUE
x <- c(as(2,"mpfr") ^ 100, 3, 3.2, 1000000, 2^40)
is.whole(x) # one FALSE, only
```

---

log1mexp

---

*Compute  $f(a) = \log(1 \pm \exp(-a))$  Numerically Optimally*


---

**Description**

Compute  $f(a) = \log(1 - \exp(-a))$ , respectively  $g(x) = \log(1 + \exp(x))$  quickly numerically accurately.

**Usage**

```
log1mexp(a, cutoff = log(2))
log1pexp(x, c0 = -37, c1 = 18, c2 = 33.3)
```

**Arguments**

a	numeric (or <code>mpfr</code> ) vector of positive values.
x	numeric vector, may also be an "mpfr" object.
cutoff	positive number; $\log(2)$ is "optimal", but the exact value is unimportant, and anything in $[0.5, 1]$ is fine.
c0, c1, c2	cutoffs for <code>log1pexp</code> ; see below.

**Value**

$$\log1mexp(a) := f(a) = \log(1 - \exp(-a)) = \log1p(-\exp(-a)) = \log(-\expm1(-a))$$

or, respectively,

$$\log1pexp(x) := g(x) = \log(1 + \exp(x)) = \log1p(\exp(x))$$

computed accurately and quickly.

**Author(s)**

Martin Maechler, May 2002; `log1pexp()` in 2012

**References**

Martin Mächler (2012). Accurately Computing  $\log(1 - \exp(-|a|))$ ; <https://CRAN.R-project.org/package=Rmpfr/vignettes/log1mexp-note.pdf>.

**Examples**

```
fExpr <- expression(
  DEF = log(1 - exp(-a)),
  expm1 = log(-expm1(-a)),
  log1p = log1p(-exp(-a)),
  F = log1mexp(a))
a. <- 2^seq(-58, 10, length = 256)
a <- a. ; str(fa <- do.call(cbind, as.list(fExpr)))
head(fa)# expm1() works here
tail(fa)# log1p() works here

## graphically:
lwd <- 1.5*(5:2); col <- adjustcolor(1:4, 0.4)
op <- par(mfcol=c(1,2), mgp = c(1.25, .6, 0), mar = .1+c(3,2,1,1))
matplot(a, fa, type = "l", log = "x", col=col, lwd=lwd)
legend("topleft", fExpr, col=col, lwd=lwd, lty=1:4, bty="n")
# expm1() & log1mexp() work here

matplot(a, -fa, type = "l", log = "xy", col=col, lwd=lwd)
legend("left", paste("-",fExpr), col=col, lwd=lwd, lty=1:4, bty="n")
# log1p() & log1mexp() work here
par(op)
```



```

aM <- 2^seqMpfr(-58, 10, length=length(a.)) # => default prec = 128
a <- aM; dim(faM <- do.call(cbind, as.list(fExpr))) # 256 x 4, "same" as 'fa'
## Here, for small 'a' log1p() and even 'DEF' is still good enough
l_f <- asNumeric(log(-faM))
all.equal(l_f[, "F"], l_f[, "log1p"], tol=0) # see TRUE (Lnx 64-bit)
io <- a. < 80 # for these, the differences are small
all.equal(l_f[io, "F"], l_f[io, "expm1"], tol=0) # see 6.662e-9
all.equal(l_f[io, "F"], l_f[io, "DEF" ], tol=0)
stopifnot(exprs = {
  all.equal(l_f[, "F"], l_f[, "log1p"],      tol= 1e-15)
  all.equal(l_f[io, "F"], l_f[io, "expm1"],  tol= 1e-7)
  all.equal(l_f[io, "F"], l_f[io, "DEF" ],  tol= 1e-7)
})
## For 128-bit prec, if we go down to 2^-130, "log1p" is no longer ok:
aM2 <- 2^seqMpfr(-130, 10, by = 1/2)
a <- aM2; fa2 <- do.call(cbind, as.list(fExpr))
head(asNumeric(fa2), 12)
tail(asNumeric(fa2), 12)

matplot(a, log(-fa2[,1:3]) -log(-fa2[, "F"]), type="l", log="x",
        lty=1:3, lwd=2*(3:1)-1, col=adjustcolor(2:4, 1/3))
legend("top", colnames(fa2)[1:3], lty=1:3, lwd=2*(3:1)-1, col=adjustcolor(2:4, 1/3))

cols <- adjustcolor(2:4, 1/3); lwd <- 2*(3:1)-1
matplot(a, 1e-40+abs(log(-fa2[,1:3]) -log(-fa2[, "F"])), type="o", log="xy",
        main = "log1mexp(a) -- approximation rel.errors, mpfr(*, prec=128)",
        pch=21:23, cex=.6, bg=5:7, lty=1:2, lwd=lwd, col=cols)
legend("top", colnames(fa2)[1:3], bty="n", lty=1:2, lwd=lwd, col=cols,
        pch=21:23, pt.cex=.6, pt.bg=5:7)

## ----- log1pexp() [simpler] -----

curve(log1pexp, -10, 10, asp=1)
abline(0,1, h=0,v=0, lty=3, col="gray")

## Cutoff c1 for log1pexp() -- not often "needed":
curve(log1p(exp(x)) - log1pexp(x), 16, 20, n=2049)
## need for *some* cutoff:
x <- seq(700, 720, by=2)
cbind(x, log1p(exp(x)), log1pexp(x))

## Cutoff c2 for log1pexp():
curve((x+exp(-x)) - x, 20, 40, n=1025)
curve((x+exp(-x)) - x, 33.1, 33.5, n=1025)

```

**Description**

Matrix / vector multiplication of `mpfr` (and “simple” `numeric`) matrices and vectors.

`matmult(x,y, fPrec = 2)` or `crossprod(x,y, fPrec = 2)` use higher precision in underlying computations.

**Usage**

```
matmult(x, y, ...)
```

**Arguments**

<code>x, y</code>	<code>numeric</code> or <code>mpfrMatrix</code> -classed <code>R</code> objects, i.e. semantically numeric matrices or vectors.
<code>...</code>	arguments passed to the hidden underlying <code>.matmult.R()</code> work horse which is also underlying the <code>%%</code> , <code>crossprod()</code> , and <code>tcrossprod()</code> methods, see the <code>mpfrMatrix</code> class documentation:
	<b>fPrec</b> a multiplication factor, a positive number determining the number of bits <code>precBits</code> used for the underlying multiplication and summation arithmetic. The default is <code>fPrec = 1</code> . Setting <code>fPrec = 2</code> doubles the precision which has been recommended, e.g., by John Nash.
	<b>precBits</b> the number of bits used for the underlying multiplication and summation arithmetic; by default <code>precBits = fPrec * max(getPrec(x), getPrec(y))</code> which typically uses the same accuracy as regular <code>mpfr</code> -arithmetic would use.

**Value**

a (base `R`) `matrix` or `mpfrMatrix`, depending on the classes of `x` and `y`.

**Note**

Using `matmult(x,y)` instead of `x %*% y`, makes sense mainly *if* you use non-default `fPrec` or `precBits` arguments.

The `crossprod()`, and `tcrossprod()` function have the *identical* optional arguments `fPrec` or `precBits`.

**Author(s)**

Martin Maechler

**See Also**

`%%`, `crossprod`, `tcrossprod`.

## Examples

```
## FIXME: add example

## 1) matmult() <--> %**%

## 2) crossprod() , tcrossprod() %*% <--> ./mpfrMatrix-class.Rd examples (!)
```

---

Mnumber-class	<i>Class "Mnumber" and "mNumber" of "mpfr" and regular numbers and arrays from them</i>
---------------	---

---

## Description

Classes "Mnumber" "mNumber" are class unions of "mpfr" and regular numbers and arrays from them.

Its purpose is for method dispatch, notably defining a `cbind(...)` method where ... contains objects of one of the member classes of "Mnumber".

Classes "mNumber" is considerably smaller is it does *not* contain "matrix" and "array" since these also extend "character" which is not really desirable for generalized numbers. It extends the simple "numericVector" class by mpfr\* classes.

## Methods

```
%*% signature(x = "mpfrMatrix", y = "Mnumber"): ...
crossprod signature(x = "mpfr", y = "Mnumber"): ...
tcrossprod signature(x = "Mnumber", y = "mpfr"): ...
etc. These are documented with the classes mpfr and or mpfrMatrix.
```

## See Also

the [array\\_or\\_vector](#) sub class; [cbind-methods](#).

## Examples

```
## "Mnumber" encompasses (i.e., "extends") quite a few
## "vector / array - like" classes:
showClass("Mnumber")
stopifnot(extends("mpfrMatrix", "Mnumber"),
          extends("array", "Mnumber"))

Mnsub <- names(getClass("Mnumber")@subclasses)
(mNsub <- names(getClass("mNumber")@subclasses))
## mNumber has *one* subclass which is not in Mnumber:
```

```

setdiff(mNsub, Mnsub)# namely "numericVector"
## The following are only subclasses of "Mnumber", but not of "mNumber":
setdiff(Mnsub, mNsub)

```

---

mpfr

Create "mpfr" Numbers (Objects)

---

## Description

Create multiple (i.e. typically *high*) precision numbers, to be used in arithmetic and mathematical computations with R.

## Usage

```

mpfr(x, precBits, ...)
## Default S3 method:
mpfr(x, precBits, base = 10,
      rnd.mode = c("N","D","U","Z","A"), scientific = NA, ...)

Const(name = c("pi", "gamma", "catalan", "log2"), prec = 120L,
      rnd.mode = c("N","D","U","Z","A"))

is.mpfr(x)

```

## Arguments

x	a <a href="#">numeric</a> , <a href="#">mpfr</a> , <a href="#">bigz</a> , <a href="#">bigq</a> , or <a href="#">character</a> vector or <a href="#">array</a> .
precBits, prec	a number, the maximal precision to be used, in <i>bits</i> ; i.e. 53 corresponds to double precision. Must be at least 2. If <a href="#">missing</a> , <a href="#">getPrec(x)</a> determines a default precision.
base	(only when x is <a href="#">character</a> ) the base with respect to which x[i] represent numbers; base <i>b</i> must fulfill $2 \leq b \leq 62$ .
rnd.mode	a 1-letter string specifying how <i>rounding</i> should happen at C-level conversion to MPFR, see details.
scientific	(used only when x is the result of <a href="#">formatBin()</a> , i.e., of class "Bcharacter:") logical indicating that the binary representation of x is in scientific notation. When TRUE, <a href="#">mpfr()</a> will substitute 0 for _; when NA, <a href="#">mpfr()</a> will guess, and use TRUE when finding a "p" in x; see also <a href="#">formatBin</a> .
name	a string specifying the mpfrlib - internal constant computation. "gamma" is Euler's gamma ( $\gamma$ ), and "catalan" Catalan's constant.
...	potentially further arguments passed to and from methods.

## Details

The "mpfr" method of `mpfr()` is a simple wrapper around `roundMpfr()`. MPFR supports the following rounding modes,

**GMP\_RNDN:** round to nearest (roundTiesToEven in IEEE 754-2008).

**GMP\_RNDZ:** round toward zero (roundTowardZero in IEEE 754-2008).

**GMP\_RNDU:** round toward plus infinity ("Up", roundTowardPositive in IEEE 754-2008).

**GMP\_RNDD:** round toward minus infinity ("Down", roundTowardNegative in IEEE 754-2008).

**GMP\_RNDA:** round away from zero (new since MPFR 3.0.0).

The 'round to nearest' ("N") mode, the default here, works as in the IEEE 754 standard: in case the number to be rounded lies exactly in the middle of two representable numbers, it is rounded to the one with the least significant bit set to zero. For example, the number  $5/2$ , which is represented by (10.1) in binary, is rounded to (10.0)=2 with a precision of two bits, and not to (11.0)=3. This rule avoids the "drift" phenomenon mentioned by Knuth in volume 2 of *The Art of Computer Programming* (Section 4.2.2).

When `x` is `character`, `mpfr()` will detect the precision of the input object.

## Value

an object of (S4) class `mpfr`, or for `mpfr(x)` when `x` is an array, `mpfrMatrix`, or `mpfrArray` which the user should just as a normal numeric vector or array.

`is.mprfr()` returns TRUE or FALSE.

## Author(s)

Martin Maechler

## References

The MPFR team. (201x). *GNU MPFR – The Multiple Precision Floating-Point Reliable Library*; see <https://www.mpfr.org/mpfr-current/#doc> or directly <https://www.mpfr.org/mpfr-current/mpfr.pdf>.

## See Also

The class documentation `mpfr` contains more details. Use `asNumeric` to transform back to double precision ("`numeric`").

## Examples

```
mpfr(pi, 120) ## the double-precision pi "translated" to 120-bit precision

pi. <- Const("pi", prec = 260) # pi "computed" to correct 260-bit precision
pi. # nicely prints 80 digits [260 * log10(2) ~= 78.3 ~ 80]

Const("gamma", 128L) # 0.5772...
Const("catalan", 128L) # 0.9159...
```

```

x <- mpfr(0:7, 100)/7 # a more precise version of k/7, k=0,...,7
x
1 / x

## character input :
mpfr("2.718281828459045235360287471352662497757") - exp(mpfr(1, 150))
## ~= -4 * 10^-40
## Also works for NA, NaN, ... :
cx <- c("1234567890123456", 345, "NA", "NaN", "Inf", "-Inf")
mpfr(cx)

## with some 'base' choices :
print(mpfr("111.1111", base=2)) * 2^4

mpfr("af21.01020300a0b0c", base=16)
## 68 bit prec. 44833.00393694653820642

mpfr("ugi0", base = 32) == 10^6 ## TRUE

## --- Large integers from package 'gmp':
Z <- as.bigz(7)^(1:200)
head(Z, 40)
## mfpr(Z) by default chooses the correct *maximal* default precision:
mZ. <- mpfr(Z)
## more efficiently chooses precision individually
m.Z <- mpfr(Z, precBits = frexpZ(Z)$exp)
## the precBits chosen are large enough to keep full precision:
stopifnot(identical(cZ <- as.character(Z),
                    as(mZ., "character")),
           identical(cZ, as(m.Z, "character")))

## compare mpfr-arithmetic with exact rational one:
stopifnot(all.equal(mpfr(as.bigq(355,113), 99),
                    mpfr(355, 99) / 113, tol = 2^-98))

## look at different "rounding modes":
sapply(c("N", "D", "U", "Z", "A"), function(RND)
  mpfr(c(-1,1)/5, 20, rnd.mode = RND), simplify=FALSE)

symnum(sapply(c("N", "D", "U", "Z", "A"),
              function(RND) mpfr(0.2, prec = 5:15, rnd.mode = RND) < 0.2 ))

```

---

mpfr-class

---

Class "mpfr" of Multiple Precision Floating Point Numbers

---

## Description

"mpfr" is the class of **M**ultiple **P**recision **F**loatingpoint numbers with **R**eliable arithmetic.

For the high-level user, "mpfr" objects should behave as standard R's `numeric` vectors. They would just print differently and use the prespecified (typically high) precision instead of the double precision of 'traditional' R numbers (with `class(.) == "numeric"` and `typeof(.) == "double"`). `hypot(x, y)` computes the hypotenuse length  $z$  in a rectangular triangle with "leg" side lengths  $x$  and  $y$ , i.e.,

$$z = \text{hypot}(x, y) = \sqrt{x^2 + y^2},$$

in a numerically stable way.

### Usage

```
hypot(x, y, rnd.mode = c("N", "D", "U", "Z", "A"))
```

### Arguments

<code>x, y</code>	an object of class <code>mpfr</code> .
<code>rnd.mode</code>	a 1-letter string specifying how <i>rounding</i> should happen at C-level conversion to MPFR, see <code>mpfr</code> .

### Objects from the Class

Objects are typically created by `mpfr(<number>, precBits)`.

`summary(<mpfr>)` returns an object of class "summaryMpfr" which contains "mpfr" but has its own `print` method.

### Slots

Internally, "mpfr" objects just contain standard R `lists` where each list element is of class "mpfr1", representing *one* MPFR number, in a structure with four slots, very much parallelizing the C `struc` in the `mpfr` C library to which the **Rmpfr** package interfaces.

An object of class "mpfr1" has slots

`prec`: "integer" specifying the maximal precision in **bits**.

`exp`: "integer" specifying the base-2 exponent of the number.

`sign`: "integer", typically -1 or 1, specifying the sign (i.e. `sign(.)`) of the number.

`d`: an "integer" vector (of 32-bit "limbs") which corresponds to the full mantissa of the number.

### Methods

**abs** signature(`x = "mpfr"`): ...

**atan2** signature(`y = "mpfr"`, `x = "ANY"`), and

**atan2** signature(`x = "ANY"`, `y = "mpfr"`): compute the arc-tangent of two arguments: `atan2(y, x)` returns the angle between the x-axis and the vector from the origin to  $(x, y)$ , i.e., for positive arguments `atan2(y, x) == atan(y/x)`.

**lbeta** signature(`a = "ANY"`, `b = "mpfrArray"`), is  $\log(|B(a, b)|)$  where  $B(a, b)$  is the Beta function, `beta(a, b)`.

**beta** signature(`a = "mpfr"`, `b = "ANY"`),

**beta** signature( $a = \text{"mpfr"}, b = \text{"mpfr"}), \dots$ , etc: Compute the beta function  $B(a, b)$ , using high precision, building on internal [gamma](#) or [lgamma](#). See the help for R's base function [beta](#) for more. Currently, there,  $a, b \geq 0$  is required. Here, we provide (non-NaN) for all numeric  $a, b$ .

When either  $a, b$ , or  $a + b$  is a negative *integer*,  $\Gamma(\cdot)$  has a pole there and is undefined (NaN). However the Beta function can be defined there as "limit", in some cases. Following other software such as SAGE, Maple or Mathematica, we provide finite values in these cases. However, note that these are not proper limits (two-dimensional in  $(a, b)$ ), but useful for some applications. E.g.,  $B(a, b)$  is defined as zero when  $a + b$  is a negative integer, but neither  $a$  nor  $b$  is. Further, if  $a > b > 0$  are integers,  $B(-a, b) = B(b, -a)$  can be seen as  $(-1)^b * B(a - b + 1, b)$ .

**dim<-** signature( $x = \text{"mpfr"})$ : Setting a dimension **dim** on an "mpfr" object makes it into an object of class "[mpfrArray](#)" or (more specifically) "[mpfrMatrix](#)" for a length-2 dimension, see their help page; note that  $t(x)$  (below) is a special case of this.

**Ops** signature( $e1 = \text{"mpfr"}, e2 = \text{"ANY"})$ : ...

**Ops** signature( $e1 = \text{"ANY"}, e2 = \text{"mpfr"})$ : ...

**Arith** signature( $e1 = \text{"mpfr"}, e2 = \text{"missing"})$ : ...

**Arith** signature( $e1 = \text{"mpfr"}, e2 = \text{"mpfr"})$ : ...

**Arith** signature( $e1 = \text{"mpfr"}, e2 = \text{"integer"})$ : ...

**Arith** signature( $e1 = \text{"mpfr"}, e2 = \text{"numeric"})$ : ...

**Arith** signature( $e1 = \text{"integer"}, e2 = \text{"mpfr"})$ : ...

**Arith** signature( $e1 = \text{"numeric"}, e2 = \text{"mpfr"})$ : ...

**Compare** signature( $e1 = \text{"mpfr"}, e2 = \text{"mpfr"})$ : ...

**Compare** signature( $e1 = \text{"mpfr"}, e2 = \text{"integer"})$ : ...

**Compare** signature( $e1 = \text{"mpfr"}, e2 = \text{"numeric"})$ : ...

**Compare** signature( $e1 = \text{"integer"}, e2 = \text{"mpfr"})$ : ...

**Compare** signature( $e1 = \text{"numeric"}, e2 = \text{"mpfr"})$ : ...

**Logic** signature( $e1 = \text{"mpfr"}, e2 = \text{"mpfr"})$ : ...

**Summary** signature( $x = \text{"mpfr"})$ : The S4 [Summary](#) group functions, [max](#), [min](#), [range](#), [prod](#), [sum](#), [any](#), and [all](#) are all defined for MPFR numbers. [mean\(x, trim\)](#) for non-0 trim works analogously to [mean.default](#).

**median** signature( $x = \text{"mpfr"})$ : works via

**quantile** signature( $x = \text{"mpfr"})$ : a simple wrapper of the [quantile.default](#) method from [stats](#).

**summary** signature( $object = \text{"mpfr"})$ : modeled after [summary.default](#), ensuring to provide the full "mpfr" range of numbers.

**Math** signature( $x = \text{"mpfr"})$ : All the S4 [Math](#) group functions are defined, using multiple precision (MPFR) arithmetic, from [getGroupMembers\("Math"\)](#), these are (in alphabetical order): [abs](#), [sign](#), [sqrt](#), [ceiling](#), [floor](#), [trunc](#), [cummax](#), [cummin](#), [cumprod](#), [cumsum](#), [exp](#), [expm1](#), [log](#), [log10](#), [log2](#), [log1p](#), [cos](#), [cosh](#), [sin](#), [sinh](#), [tan](#), [tanh](#), [acos](#), [acosh](#), [asin](#), [asinh](#), [atan](#), [atanh](#), [cospi](#), [sinpi](#), [tanpi](#), [gamma](#), [lgamma](#), [digamma](#), and [trigamma](#).

Currently, [trigamma](#) is not provided by the MPFR library and hence not yet implemented. Further, the [cum\\*\(\)](#) methods are *not yet* implemented.



**factorial** signature(x = "mpfr"): this will `round` the result when x is integer valued. Note however that `factorialMpfr(n)` for integer n is slightly more efficient, using the MPFR function 'mpfr\_fac\_ui'.

**Math2** signature(x = "mpfr"): `round(x, digits)` and `signif(x, digits)` methods. Note that these do not change the formal precision ('prec' slot), and you may often want to apply `roundMpfr()` in addition or preference.

**as.numeric** signature(x = "mpfr"): ...

**as.vector** signature(x = "mpfrArray"): as for standard `arrays`, this "drops" the dim (and dimnames), i.e., transforms x into an 'MPFR' number vector, i.e., class `mpfr`.

`[[` signature(x = "mpfr", i = "ANY"), and

`[` signature(x = "mpfr", i = "ANY", j = "missing", drop = "missing"): subsetting aka "indexing" happens as for numeric vectors.

**format** signature(x = "mpfr"), further arguments `digits = NULL`, `scientific = NA`, etc: returns `character` vector of same length as x; when `digits` is `NULL`, with *enough* digits to recreate x accurately. For details, see `formatMpfr`.

**is.finite** signature(x = "mpfr"): ...

**is.infinite** signature(x = "mpfr"): ...

**is.na** signature(x = "mpfr"): ...

**is.nan** signature(x = "mpfr"): ...

**log** signature(x = "mpfr"): ...

**show** signature(object = "mpfr"): ...

**sign** signature(x = "mpfr"): ...

**Re, Im** signature(z = "mpfr"): simply return z or 0 (as "mpfr" numbers of correct precision), as mpfr numbers are 'real' numbers.

**Arg, Mod, Conj** signature(z = "mpfr"): these are trivial for our 'real' mpfr numbers, but defined to work correctly when used in R code that also allows complex number input.

**all.equal** signature(target = "mpfr", current = "mpfr"),

**all.equal** signature(target = "mpfr", current = "ANY"), and

**all.equal** signature(target = "ANY", current = "mpfr"): methods for numerical (approximate) equality, `all.equal` of multiple precision numbers. Note that the default tolerance (argument) is taken to correspond to the (smaller of the two) precisions when both main arguments are of class "mpfr", and hence can be considerably less than double precision machine epsilon `.Machine$double.eps`.

**coerce** signature(from = "numeric", to = "mpfr"): `as(., "mpfr")` coercion methods are available for `character` strings, `numeric`, `integer`, `logical`, and even `raw`. Note however, that `mpfr(., precBits, base)` is more flexible.

**coerce** signature(from = "mpfr", to = "bigz"): coerces to biginteger, see `bigz` in package `gmp`.

**coerce** signature(from = "mpfr", to = "numeric"): ...

**coerce** signature(from = "mpfr", to = "character"): ...

**unique** signature(x = "mpfr"), and corresponding S3 method (such that `unique(<mpfr>)` works inside `base` functions), see `unique`.

Note that `duplicated()` works for "mpfr" objects without the need for a specific method.

**t** signature( $x = \text{"mpfr"}$ ): makes  $x$  into an  $n \times 1$  `mpfrMatrix`.

**which.min** signature( $x = \text{"mpfr"}$ ): gives the index of the first minimum, see [which.min](#).

**which.max** signature( $x = \text{"mpfr"}$ ): gives the index of the first maximum, see [which.max](#).

### Note

Many more methods (“functions”) automagically work for “mpfr” number vectors (and matrices, see the `mpfrMatrix` class doc), notably [sort](#), [order](#), [quantile](#), [rank](#).

### Author(s)

Martin Maechler

### See Also

The “`mpfrMatrix`” class, which extends the “mpfr” one.

[roundMpfr](#) to *change* precision of an “mpfr” object which is typically desirable *instead* of or in addition to `signif()` or `round()`; [is.whole\(\) etc.](#)

Special mathematical functions such as some Bessel ones, e.g., [jn](#); further, [zeta\(.\)](#) ( $= \zeta(.)$ ), [Ei\(\)](#) etc. [Bernoulli](#) numbers and the Pochhammer function [pochMpfr](#).

### Examples

```
## 30 digit precision
(x <- mpfr(c(2:3, pi), prec = 30 * log2(10)))
str(x) # str() displays *compact*ly => not full precision
x^2
x[1] / x[2] # 0.66666... ~ 30 digits

## indexing - as with numeric vectors
stopifnot(exprs = {
  identical(x[2], x[[2]])
  ## indexing "outside" gives NA (well: "mpfr-NaN" for now):
  is.na(x[5])
  ## whereas "[[" cannot index outside:
  inherits(tryCatch(x[[5]], error=identity), "error")
  ## and only select *one* element:
  inherits(tryCatch(x[[2:3]], error=identity), "error")
})

## factorial() & lfactorial would work automagically via [l]gamma(),
## but factorial() additionally has an "mpfr" method which rounds
f200 <- factorial(mpfr(200, prec = 1500)) # need high prec.!
f200
as.numeric(log2(f200))# 1245.38 -- need precBits >~ 1246 for full precision

##--> see factorialMpfr() for more such computations.

##--- "Underflow" **much** later -- exponents have 30(+1) bits themselves:
```

```

mpfr.min.exp2 <- - (2^30 + 1)
two <- mpfr(2, 55)
stopifnot(two ^ mpfr.min.exp2 == 0)
## whereas
two ^ (mpfr.min.exp2 * (1 - 1e-15))
## 2.38256490488795107e-323228497 ["typically"]

##--- "Assert" that {sort}, {order}, {quantile}, {rank}, all work :

p <- mpfr(rpois(32, lambda=500), precBits=128)^10
np <- as.numeric(log(p))
(sp <- summary(p))# using the print.summaryMpfr() method
stopifnot(all(diff(sort(p)) >= 0),
  identical(order(p), order(np)),
  identical(rank(p), rank(np)),
  all.equal(sapply(1:9, function(Typ) quantile(np, type=Typ, names=FALSE)),
    sapply(lapply(1:9, function(Typ) quantile(p, type=Typ, names=FALSE)),
      function(x) as.numeric(log(x))),
    tol = 1e-3),# quantiles: interpolated in orig. <--> log scale
TRUE)

m0 <- mpfr(numeric(), 99)
xy <- expand.grid(x = -2:2, y = -2:2) ; x <- xy[, "x"] ; y <- xy[, "y"]
a2. <- atan2(y,x)

stopifnot(identical(which.min(m0), integer(0)),
  identical(which.max(m0), integer(0)),
  all.equal(a2., atan2(as(y,"mpfr"), x)),
  max(m0) == mpfr(-Inf, 53), # (53 is not a feature, but ok)
  min(m0) == mpfr(+Inf, 53),
  sum(m0) == 0, prod(m0) == 1)

## unique(), now even base::factor() "works" on <mpfr> :
set.seed(17)
p <- rlnorm(20) * mpfr(10, 100)^-999
pp <- sample(p, 50, replace=TRUE)
str(unique(pp)) # length 18 .. (from originally 20)
## Class 'mpfr' [package "Rmpfr"] of length 18 and precision 100
## 5.56520587824e-999 4.41636588227e-1000 ..
facp <- factor(pp)
str(facp) # the factor *levels* are a bit verbose :
# Factor w/ 18 levels "new(\"mpfr1\", ....." ..
# At least *some* factor methods work :
stopifnot(exprs = {
  is.factor(facp)
  identical(unname(table(facp)),
    unname(table(asNumeric(pp * mpfr(10,100)^1000))))
})

## ((unfortunately, the expressions are wrong; should integer "L"))
#
## More useful: levels with which to *invert* factor() :
## -- this is not quite ok:

```

```
## simplified from 'utils' :
deparse1 <- function(x, ...) paste(deparse(x, 500L, ...), collapse = " ")
if(FALSE) {
  str(pp.levs <- vapply(unclass(sort(unique(pp))), deparse1, ""))
  facp2 <- factor(pp, levels = pp.levs)
}
```

---

mpfr-distr-etc

*Distribution Functions etc (MPFR)*


---

## Description

For some R standard (probability) density, distribution or quantile functions, we provide MPFR versions.

## Usage

```
dpois(x, lambda, log = FALSE, useLog = )
dbinom(x, size, prob, log = FALSE, useLog = )
dnbinom(x, size, prob, mu, log = FALSE, useLog = any(x > 1e6))
dnorm(x, mean = 0, sd = 1, log = FALSE)
dgamma(x, shape, rate = 1, scale = 1/rate, log = FALSE)
dt(x, df, ncp, log = FALSE)

pnorm(q, mean = 0, sd = 1, lower.tail = TRUE, log.p = FALSE)
```

## Arguments

`x, q, lambda, size, prob, mu, mean, sd, shape, rate, scale, df, ncp`  
**numeric** or **mpfr** vectors. All of these are “recycled” to the length of the longest one. For their meaning/definition, see the corresponding standard R (**stats** package) function.

`log, log.p, lower.tail`  
**logical**, see **pnorm**, **dpois**, etc.

`useLog`  
**logical** with default depending on `x` etc, indicating if log-scale computation should be used even when `log = FALSE`, for performance or against overflow / underflow.

## Details

`pnorm()` is based on `erf()` and `erfc()` which have direct MPFR counter parts and are both reparametrizations of `pnorm`,  $\text{erf}(x) = 2 * \text{pnorm}(\sqrt{2} * x)$  and  $\text{erfc}(x) = 2 * \text{pnorm}(\sqrt{2} * x, \text{lower} = \text{FALSE})$ .

## Value

A vector of the same length as the longest of `x, q, ...`, of class **mpfr** with the high accuracy results of the corresponding standard R function.

**Note**

E.g., for `pnorm(*, log.p = TRUE)` to be useful, i.e., not to underflow or overflow, you may want to extend the exponential range of MPFR numbers, using `.mpfr_erange_set()`, see the examples.

**See Also**

`pnorm`, `dt`, `dbinom`, `dnbinom`, `dgamma`, `dpois` in standard package `stats`.

`pbetaI(x, a, b)` is a `mpfr` version of `pbeta` only for *integer* `a` and `b`.

**Examples**

```
x <- 1400+ 0:10
print(dpois(x, 1000), digits =18) ## standard R's double precision
(px <- dpois(mpfr(x, 120), 1000))## more accuracy for the same
px. <- dpois(mpfr(x, 120), 1000, useLog=TRUE)# {failed in 0.8-8}
stopifnot(all.equal(px, px., tol = 1e-31))
dpois(0:5, mpfr(10000, 80)) ## very small exponents (underflowing in dbl.prec.)

print(dbinom(0:8, 8, pr = 4 / 5), digits=18)
  dbinom(0:8, 8, pr = 4/mpfr(5, 99)) -> dB; dB

print(dnorm(      -5:5), digits=18)
  dnorm(mpfr(-5:5, prec=99))

## For pnorm() in the extreme tails, need an exponent range
## larger than the (MPFR and Rmpfr) default:
(old_eranges <- .mpfr_erange()) # typically +/- 2^30
.mpfr_erange_set(value = (1-2^-52)*.mpfr_erange(c("min.emin", "max.emax")))
tens <- mpfr(10^(4:7), 128)
pnorm(tens, lower.tail=FALSE, log.p=TRUE) # "works"
## reset to previous
.mpfr_erange_set( , old_eranges)
pnorm(tens, lower.tail=FALSE, log.p=TRUE) # all but first underflow to -Inf
```

---

mpfr-special-functions

*Special Mathematical Functions (MPFR)*

---

**Description**

Special Mathematical Functions, supported by the MPFR Library.

Note that additionally, all the `Math` and `Math2` group member functions are “mpfr-ified”, too; ditto, for many more standard R functions. See the methods listed in `mpfr` (aka `?`mpfr-class``).

**Usage**

zeta(x)  
 Ei(x)  
 Li2(x)  
  
 erf(x)  
 erfc(x)

**Arguments**

x                    a `numeric` or `mpfr` vector.

**Details**

zeta(x) computes Riemann's Zeta function  $\zeta(x)$  important in analytical number theory and related fields. The traditional definition is

$$\zeta(x) = \sum_{n=1}^{\infty} \frac{1}{n^x}.$$

Ei(x) computes the exponential integral,

$$\int_{-\infty}^x \frac{e^t}{t} dt.$$

Li2(x) computes the dilogarithm,

$$\int_0^x \frac{-\log(1-t)}{t} dt.$$

erf(x) and erfc(x) are the error, respectively complementary error function which are both reparametrizations of `pnorm`,  $\text{erf}(x) = 2 * \text{pnorm}(\sqrt{2} * x)$  and  $\text{erfc}(x) = 2 * \text{pnorm}(\sqrt{2} * x, \text{lower}=\text{FALSE})$ , and hence **Rmpfr** provides its own version of `pnorm`.

**Value**

A vector of the same length as x, of class `mpfr`.

**See Also**

`pnorm` in standard package **stats**; the class description `mpfr` mentioning the generic arithmetic and mathematical functions (`sin`, `log`, ..., etc) for which "mpfr" methods are available.

Note the (integer order, non modified) Bessel functions  $j_0()$ ,  $y_n()$ , etc, named `j0`, `yn` etc, and Airy function  $Ai()$  in `Bessel_mpfr`.

**Examples**

```

curve(Ei, 0, 5, n=2001)

## As we now require (mpfrVersion() >= "2.4.0"):
curve(Li2, 0, 5, n=2001)
curve(Li2, -2, 13, n=2000); abline(h=0,v=0, lty=3)
curve(Li2, -200,400, n=2000); abline(h=0,v=0, lty=3)

curve(erf, -3,3, col = "red", ylim = c(-1,2))
curve(erfc, add = TRUE, col = "blue")
abline(h=0, v=0, lty=3)
legend(-3,1, c("erf(x)", "erfc(x)"), col = c("red","blue"), lty=1)

```

mpfr-utils

*Rmpfr – Utilities for Precision Setting, Printing, etc***Description**

This page documents utilities from package **Rmpfr** which are typically not called by the user, but may come handy in some situations.

Notably, the (base-2) maximal (and minimal) precision and the “erange”, the range of possible (base-2) exponents of `mpfr`-numbers can be queried and partly extended.

**Usage**

```

getPrec(x, base = 10, doNumeric = TRUE, is.mpfr = NA, bigq. = 128L)
.getPrec(x)
getD(x)
mpfr_default_prec(prec)
## S3 method for class 'mpfrArray'
print(x, digits = NULL, drop0trailing = FALSE,
      right = TRUE,
      max.digits = getOption("Rmpfr.print.max.digits", 999L),
      exponent.plus = getOption("Rmpfr.print.exponent.plus", TRUE),
      ...)
## S3 method for class 'mpfr'
print(x, digits = NULL, drop0trailing = TRUE,
      right = TRUE,
      max.digits = getOption("Rmpfr.print.max.digits", 999L),
      exponent.plus = getOption("Rmpfr.print.exponent.plus", TRUE),
      ...)
toNum(from, rnd.mode = c('N','D','U','Z','A'))
mpfr2array(x, dim, dimnames = NULL, check = FALSE)

.mpfr2list(x, names = FALSE)

mpfrXport(x, names = FALSE)

```

```

mpfrImport(mxp)

.mpfr_formatinfo(x)
.mpfr2exp(x)

.mpfr_erange(kind = c("Emin", "Emax"), names = TRUE)
.mpfr_erange_set(kind = c("Emin", "Emax"), value)
.mpfr_erange_kinds
.mpfr_erange_is_int()
.mpfr_maxPrec()
.mpfr_minPrec()

```

### Arguments

<code>x</code> , <code>from</code>	typically, an R object of class "mpfr", or "mpfrArray", respectively. For <code>getPrec()</code> , any number-like R object, or <code>NULL</code> .
<code>base</code>	(only when <code>x</code> is <code>character</code> ) the base with respect to which <code>x[i]</code> represent numbers; base $b$ must fulfill $2 \leq b \leq 62$ .
<code>doNumeric</code>	logical indicating <code>integer</code> or <code>double</code> typed <code>x</code> should be accepted and a default precision be returned. Should typically be kept at default <code>TRUE</code> .
<code>is.mpfr</code>	logical indicating if <code>class(x)</code> is already known to be "mpfr"; typically should be kept at default, <code>NA</code> .
<code>bigq</code> .	for <code>getPrec()</code> , the precision to use for a big rational (class "bigq"); if not specified gives warning when used.
<code>prec</code>	a positive integer, or missing.
<code>drop0trailing</code>	logical indicating if trailing "0"s should be omitted.
<code>right</code>	logical indicating <code>print()</code> ing should right justify the strings; see <code>print.default()</code> to which it is passed.
<code>digits</code> , ...	further arguments to print methods.
<code>max.digits</code>	a number (possibly <code>Inf</code> ) to limit the number of (mantissa) digits to be printed, simply passed to <code>formatMpfr()</code> . The default is finite to protect from printing very long strings which is often undesirable, notably in <code>interactive</code> use.
<code>exponent.plus</code>	logical, simply passed to <code>formatMpfr()</code> . Was <code>FALSE</code> hardwired in <code>Rmpfr</code> versions before 0.8-0, and hence is allowed to be tweaked by an <code>options()</code> setting.
<code>rnd.mode</code>	a 1-letter string specifying how <i>rounding</i> should happen at C-level conversion to MPFR, see details of <code>mpfr</code> .
<code>dim</code> , <code>dimnames</code>	for "mpfrArray" construction.
<code>check</code>	logical indicating if the <code>mpfrArray</code> construction should happen with internal safety check. Previously, the implicit default used to be <code>true</code> .
<code>names</code>	(for <code>.mpfr2list()</code> ) <code>logical</code> or <code>character</code> vector, indicating if the list returned should have <code>names</code> . If <code>character</code> , it specifies the names; if <code>true</code> , the names are set to <code>format(x)</code> .
<code>mxp</code>	an "mpfrXport" object, as resulting from <code>mpfrXport()</code> .



kind	a <a href="#">character</a> string or vector, specifying the kind of “erange” value; must be an element of <code>.mpfr_erange_kinds</code> , i.e., one of "Emin", "Emax", "min.emin", "max.emin", "min.emax", "max.emax".
value	<a href="#">numeric</a> , for <code>.mpfr_erange_set()</code> one number per kind. Must be in range specified by the <code>*</code> . "emin" and <code>*</code> . "emax" erange values.

## Details

The `print` method is currently built on the `format` method for class `mpfr`. This, currently does *not* format columns jointly which leads to suboptimally looking output. There are plans to change this.

Note that `formatMpfr()` which is called by `print()` (or `show()` or R’s implicit printing) uses `max.digits = Inf`, differing from our `print()`’s default on purpose. If you do want to see the full accuracy even in cases it is large, use `options(Rmpfr.print.max.digits = Inf)` or `(. = 1e7)`, say.

The `.mpfr_erange*` functions (and variable) allow to query and set the allowed range of values for the base-2 *exponents* of "mpfr" numbers. See the examples below and GNU MPFR library documentation on the C functions `mpfr_get_emin()`, `mpfr_set_emin(.)`, `mpfr_get_emin_min()`, and `mpfr_get_emin_max()`, (and those four with ‘\_emin’ replaced by ‘\_emax’ above).

## Value

`getPrec(x)` returns a [integer](#) vector of length one or the same length as `x` when that is positive, whereas `getPrec(NULL)` returns `mpfr_default_prec()`, see below.

If you need to *change* the precision of `x`, i.e., need something like “setPrec”, use `roundMpfr()`.

`.getPrec(x)` is a simplified version of `getPrec()` which only works for "mpfr" objects `x`.

`getD(x)` is intended to be a fast version of `x@.Data`, and should not be used outside of lower level functions.

`mpfr_default_prec()` returns the current MPFR default precision, an [integer](#). This is currently not made use of much in package **Rmpfr**, where functions have their own default precision where needed, and otherwise we’d rather not be dependent of such a *global* setting.

`mpfr_default_prec(prec)` *sets* the current MPFR default precision and returns the previous one; see above.

`.mpfr_maxPrec()` and (less interestingly) `.mpfr_minPrec()` give the maximal and minimal base-2 precision allowed in the current version of the MPFR library linked to by R package **Rmpfr**. The maximal precision is typically  $2^{63}$ , i.e.,

```
all.equal(.mpfr_maxPrec(), 2^63)
```

is typically true.

`toNum(m)` returns a numeric [array](#) or [matrix](#), when `m` is of class "mpfrArray" or "mpfrMatrix", respectively. It should be equivalent to `as(m, "array")` or `...` "matrix". Note that the slightly more general `asNumeric()` is preferred now.

`mpfr2array()` a slightly more flexible alternative to `dim(.) <- dd`.

`.mpfr2exp(x)` returns the base-2 (integer valued) exponents of `x`, i.e., it is the R interface to MPFR C’s `mpfr_get_exp()`. The result is [integer](#) iff `.mpfr_erange_is_int()` is true, otherwise [double](#). Note that the MPFR (4.0.1) manual says about `mpfr_get_exp()`: *The behavior for NaN, infinity or zero is undefined.*

`.mpfr_erange_is_int()` returns TRUE iff the `.mpfr_erange(c("Emin", "Emax"))` range lies inside the range of R's `integer` limits, i.e., has absolute values not larger than `.Machine$integer.max` ( $= 2^{31} - 1$ ).

`.mpfr_erange_set()` *invisibly* (see `invisible()`) returns TRUE iff the change was successful.

`.mpfr_formatinfo(x)` returns conceptually a subset of `.mpfr2str()`'s result, a list with three components

`exp`                    the base-2 exponents of `x`, identical to `.mpfr2exp(x)`.

`finite`                `logical` identical to `is.finite(x)`.

`is.0`                  `logical` indicating if the corresponding `x[i]` is zero; identical to `mpfrIs0(x)`.

(Note that `.mpfr2str(x, ..., base)$exp` is wrt base *and* is not undefined but

### Note

`mpfrXport()` and `mpfrImport()` are **experimental** and used to explore reported platform incompatibilities of `save()`d and `load()`ed "mpfr" objects between Windows and non-Windows platforms.

In other words, the format of the result of `mpfrXport()` and hence the `mxp` argument to `mpfrImport()` are considered internal, not part of the API and subject to change.

### See Also

Start using `mpfr(...)`, and compute with these numbers.

`mpfrArray(x)` is for numeric ("non-mpfr") `x`, whereas `mpfr2array(x)` is for "mpfr" classed `x`, only.

### Examples

```
getPrec(as(c(1,pi), "mpfr")) # 128 for both

(opr <- mpfr_default_prec()) ## typically 53, the MPFR system default
stopifnot(opr == (oprec <- mpfr_default_prec(70)),
          70 == mpfr_default_prec())
## and reset it:
mpfr_default_prec(opr)

## Explore behavior of rounding modes 'rnd.mode':
x <- mpfr(10,99)^512 # too large for regular (double prec. / numeric):
sapply(c("N", "D", "U", "Z", "A"), function(RM)
       sapply(list(-x,x), function(.) toNum(., RM)))
##   N          D          U          Z          A
## -Inf          -Inf -1.797693e+308 -1.797693e+308 -Inf
##  Inf 1.797693e+308          Inf 1.797693e+308  Inf

## Printing of "MPFR" matrices is less nice than R's usual matrix printing:
m <- outer(c(1, 3.14, -1024.5678), c(1, 1e-3, 10,100))
m[3,3] <- round(m[3,3])
m
mpfr(m, 50)
```

```

B6 <- mpfr2array(Bernoulli(1:6, 60), c(2,3),
                 dimnames = list(LETTERS[1:2], letters[1:3]))
B6

## Ranges of (base 2) exponents of MPFR numbers:
.mpfr_erange() # the currently active range of possible base 2 exponents:

## A factory fresh setting fulfills
.mpfr_erange(c("Emin","Emax")) == c(-1,1) * (2^30 - 1)

## There are more 'kind's, the latter 4 showing how you could change the first two :
.mpfr_erange_kinds
.mpfr_erange(.mpfr_erange_kinds)
eLimits <- .mpfr_erange(c("min.emin", "max.emin", "min.emax", "max.emax"))
## Typically true in "current" MPFR versions:
eLimits == c(-1,1, -1,1) * (2^62 - 1)

## Looking at internal representation [for power users only!]:

i8 <- mpfr(-2:5, 32)
x4 <- mpfr(c(NA, NaN, -Inf, Inf), 32)
## The output of the following depends on the GMP "numb" size
## (32 bit vs. 64 bit), and may be even more platform specific:
str( .mpfr2list(i8) )
str( .mpfr2list(x4, names = TRUE) )

str(xp4 <- mpfrXport(x4, names = TRUE))
stopifnot(identical(x4, mpfrImport(mpfrXport(x4))),
           identical(i8, mpfrImport(mpfrXport(i8))))
## FIXME, need c(.), as dim(.) "get lost":
stopifnot(identical(c(B6), mpfrImport(mpfrXport(B6))))

```

---

mpfr.utils

*MPFR Number Utilities*


---

## Description

`mpfrVersion()` returns the version of the MPFR library which **Rmpfr** is currently linked to.

`c(x,y,...)` can be used to combine MPFR numbers in the same way as regular numbers **IFF** the first argument `x` is of class `mpfr`.

`mpfrIs0(.)` uses the MPFR library in the documented way to check if (a vector of) MPFR numbers are zero. It was called `mpfr.is.0` which is strongly deprecated now.

`.mpfr.is.whole(x)` uses the MPFR library in the documented way to check if (a vector of) MPFR numbers is integer *valued*. This is equivalent to `x == round(x)`, but *not* at all to `is.integer(as(x, "numeric"))`.

You should typically rather use (the "mpfr" method of the generic function) `is.whole(x)` instead. The former name `mpfr.is.integer` is deprecated now.

**Usage**

```
mpfrVersion()
mpfrIs0(x)

## S3 method for class 'mpfr'
c(...)
## S3 method for class 'mpfr'
diff(x, lag = 1L, differences = 1L, ...)
```

**Arguments**

`x` an object of class `mpfr`.

`...` for `diff`, further `mpfr` class objects or simple numbers (`numeric` vectors) which are coerced to `mpfr` with default precision of 128 bits.

`lag, differences` for `diff()`: exact same meaning as in `diff()`'s default method, `diff.default`.

**Value**

`mpfrIs0` returns a logical vector of length `length(x)` with values `TRUE` iff the corresponding `x[i]` is an MPFR representation of zero (0).

Similarly, `.mpfr.is.whole` and `is.whole` return a logical vector of length `length(x)`.

`mpfrVersion` returns an object of S3 class `"numeric_version"`, so it can be used in comparisons.

The other functions return MPFR number (vectors), i.e., extending class `mpfr`.

**See Also**

`str.mpfr` for the `str` method. `erf` for special mathematical functions on MPFR.

The class description `mpfr` page mentions many generic arithmetic and mathematical functions for which "mpfr" methods are available.

**Examples**

```
mpfrVersion()

(x <- c(Const("pi", 64), mpfr(-2:2, 64)))
mpfrIs0(x) # one of them is
x[mpfrIs0(x)] # but it may not have been obvious..
str(x)

x <- rep(-2:2, 5)
stopifnot(is.whole(mpfr(2, 500) ^ (1:200)),
          all.equal(diff(x), diff(as.numeric(x))))
```

---

mpfrArray                      *Construct "mpfrArray" almost as by 'array()'*

---

### Description

Utility to construct an R object of class `mpfrArray`, very analogously to the numeric `array` function.

### Usage

```
mpfrArray(x, precBits, dim = length(x), dimnames = NULL,
          rnd.mode = c("N", "D", "U", "Z", "A"))
```

### Arguments

<code>x</code>	numeric(like) vector, typically of length <code>prod(dim)</code> or shorter in which case it is recycled.
<code>precBits</code>	a number, the maximal precision to be used, in <i>bits</i> ; i.e., 53 corresponds to double precision. Must be at least 2.
<code>dim</code>	the dimension of the array to be created, that is a vector of length one or more giving the maximal indices in each dimension.
<code>dimnames</code>	either NULL or the names for the dimensions. This is a list with one component for each dimension, either NULL or a character vector of the length given by <code>dim</code> for that dimension.
<code>rnd.mode</code>	a 1-letter string specifying how <i>rounding</i> should happen at C-level conversion to MPFR, see details of <code>mpfr</code> .

### Value

an object of class `"mpfrArray"`, specifically `"mpfrMatrix"` when `length(dim) == 2`.

### See Also

`mpfr`, `array`; `asNumeric()` as “inverse” of `mpfrArray()`, to get back a numeric array.  
`mpfr2array(x)` is for “mpfr” classed `x`, only, whereas `mpfrArray(x)` is for numeric (“non-mpfr”) `x`.

### Examples

```
## preallocating is possible here too
ma <- mpfrArray(NA, prec = 80, dim = 2:4)
validObject(A2 <- mpfrArray(1:24, prec = 64, dim = 2:4))

## recycles, gives an "mpfrMatrix" and dimnames :
mat <- mpfrArray(1:5, 64, dim = c(5,3), dimnames=list(NULL, letters[1:3]))
mat
asNumeric(mat)
stopifnot(identical(asNumeric(mat),
```

```

matrix(1:5 +0, 5,3, dimnames=dimnames(mat))))

## Testing the apply() method :
apply(mat, 2, range)
apply(A2, 1:2, range)
apply(A2, 2:3, max)
(fA2 <- apply(A2, 2, fivenum))
a2 <- as(A2, "array")
stopifnot(as(apply(A2, 2, range), "matrix") ==
  apply(a2, 2, range)
  , all.equal(fA2, apply(a2, 2, fivenum))
  , all.equal(apply(A2, 2, quantile),
    apply(a2, 2, quantile))
  , all.equal(A2, apply(A2, 2:3, identity) -> aA2, check.attributes=FALSE)
  , dim(A2) == dim(aA2)
)

```

---

mpfrMatrix

*Classes "mpfrMatrix" and "mpfrArray"*


---

## Description

The classes "mpfrMatrix" and "mpfrArray" are, analogously to the **base** `matrix` and `array` functions and classes simply "numbers" of class `mpfr` with an additional `Dim` and `Dimnames` slot.

## Objects from the Class

Objects should typically be created by `mpfrArray()`, but can also be created by `new("mpfrMatrix", ...)` or `new("mpfrArray", ...)`, or also by `t(x)`, `dim(x) <- dd`, or `mpfr2array(x, dim=dd)` where `x` is a an `mpfr` "number vector".

A (slightly more flexible) alternative to `dim(x) <- dd` is `mpfr2array(x, dd, dimnames)`.

## Slots

**.Data:** as for the `mpfr` class, a "list" of `mpfr1` numbers.

**Dim:** of class "integer", specifying the array dimension.

**Dimnames:** of class "list" and the same length as `Dim`, each list component either `NULL` or a `character` vector of length `Dim[j]`.

## Extends

Class "mpfrMatrix" extends "mpfrArray", directly.

Class "mpfrArray" extends class "`mpfr`", by class "mpfrArray", distance 2; class "`list`", by class "mpfrArray", distance 3; class "`vector`", by class "mpfrArray", distance 4.

## Methods

**Arith** signature(e1 = "mpfr", e2 = "mpfrArray"): ...

**Arith** signature(e1 = "numeric", e2 = "mpfrArray"): ...

**Arith** signature(e1 = "mpfrArray", e2 = "mpfrArray"): ...

**Arith** signature(e1 = "mpfrArray", e2 = "mpfr"): ...

**Arith** signature(e1 = "mpfrArray", e2 = "numeric"): ...

**as.vector** signature(x = "mpfrArray", mode = "missing"): drops the dimension 'attribute', i.e., transforms x into a simple **mpfr** vector. This is an inverse of `t(.)` or `dim(.) <- *` on such a vector.

**atan2** signature(y = "ANY", x = "mpfrArray"): ...

**atan2** signature(y = "mpfrArray", x = "mpfrArray"): ...

**atan2** signature(y = "mpfrArray", x = "ANY"): ...

`[<-` signature(x = "mpfrArray", i = "ANY", j = "ANY", value = "ANY"): ...

`[` signature(x = "mpfrArray", i = "ANY", j = "ANY", drop = "ANY"): ...

`[` signature(x = "mpfrArray", i = "ANY", j = "missing", drop = "missing"): "mpfrArray"s can be subset ("indexed") as regular R **arrays**.

`%*%` signature(x = "mpfr", y = "mpfrMatrix"): Compute the matrix/vector product  $xy$  when the dimensions (**dim**) of x and y match. If x is not a matrix, it is treated as a 1-row or 1-column matrix (aka "row vector" or "column vector") depending on which one makes sense, see the documentation of the **base** function `%*%`.

`%*%` signature(x = "mpfr", y = "Mnumber"): method definition for cases with one **mpfr** and any "number-like" argument are to use MPFR arithmetic as well.

`%*%` signature(x = "mpfrMatrix", y = "mpfrMatrix"),

`%*%` signature(x = "mpfrMatrix", y = "mpfr"), etc. Further method definitions with identical semantic.

**crossprod** signature(x = "mpfr", y = "missing"): Computes  $x'x$ , i.e., `t(x) %*% x`, typically more efficiently.

**crossprod** signature(x = "mpfr", y = "mpfrMatrix"): Computes  $x'y$ , i.e., `t(x) %*% y`, typically more efficiently.

**crossprod** signature(x = "mpfrMatrix", y = "mpfrMatrix"): ...

**crossprod** signature(x = "mpfrMatrix", y = "mpfr"): ...

**tcrossprod** signature(x = "mpfr", y = "missing"): Computes  $xx'$ , i.e., `x %*% t(x)`, typically more efficiently.

**tcrossprod** signature(x = "mpfrMatrix", y = "mpfrMatrix"): Computes  $xy'$ , i.e., `x %*% t(y)`, typically more efficiently.

**tcrossprod** signature(x = "mpfrMatrix", y = "mpfr"): ...

**tcrossprod** signature(x = "mpfr", y = "mpfrMatrix"): ...

**coerce** signature(from = "mpfrArray", to = "array"): coerces from to a *numeric* array of the same dimension.

**coerce** signature(from = "mpfrArray", to = "vector"): as for standard **arrays**, this "drops" the `dim` (and `dimnames`), i.e., returns an **mpfr** vector.

**Compare** signature(e1 = "mpfr", e2 = "mpfrArray"): ...  
**Compare** signature(e1 = "numeric", e2 = "mpfrArray"): ...  
**Compare** signature(e1 = "mpfrArray", e2 = "mpfr"): ...  
**Compare** signature(e1 = "mpfrArray", e2 = "numeric"): ...  
**dim** signature(x = "mpfrArray"): ...  
**dimnames<-** signature(x = "mpfrArray"): ...  
**dimnames** signature(x = "mpfrArray"): ...  
**show** signature(object = "mpfrArray"): ...  
**sign** signature(x = "mpfrArray"): ...  
**norm** signature(x = "mpfrMatrix", type = "character"): computes the matrix norm of x, see [norm](#) or the one in package **Matrix**.  
**t** signature(x = "mpfrMatrix"): tranpose the mpfrMatrix.  
**aperm** signature(a = "mpfrArray"): aperm(a, perm) is a generalization of t(.) to *permute* the dimensions of an mpfrArray; it has the same semantics as the standard [aperm\(\)](#) method for simple R arrays.

**Author(s)**

Martin Maechler

**See Also**

[mpfrArray](#), also for more examples.

**Examples**

```
showClass("mpfrMatrix")

validObject(mm <- new("mpfrMatrix"))
validObject(aa <- new("mpfrArray"))

v6 <- mpfr(1:6, 128)
m6 <- new("mpfrMatrix", v6, Dim = c(2L, 3L))
validObject(m6)
m6
which(m6 == 3, arr.ind = TRUE) # |--> (1, 2)
## Coercion back to "vector": Both of these work:
stopifnot(identical(as(m6, "mpfr"), v6),
  identical(as.vector(m6), v6)) # < but this is a "coincidence"

S2 <- m6[,-3] # 2 x 2
S3 <- rbind(m6, c(1:2,10)) ; s3 <- asNumeric(S3)
det(S2)
str(determinant(S2))
det(S3)
stopifnot(all.equal(det(S2), det(asNumeric(S2)), tol=1e-15),
  all.equal(det(S3), det(s3), tol=1e-15))
```



```

## 2-column matrix indexing and replacement:
(sS <- S3[i2 <- cbind(1:2, 2:3)])
stopifnot(identical(asNumeric(sS), s3[i2]))
C3 <- S3; c3 <- s3
C3[i2] <- 10:11
c3[i2] <- 10:11
stopifnot(identical(asNumeric(C3), c3))

AA <- new("mpfrArray", as.vector(cbind(S3, -S3)), Dim=c(3L,3:2))
stopifnot(identical(AA[,1] , S3), identical(AA[,2] , -S3))
aa <- asNumeric(AA)

i3 <- cbind(3:1, 1:3, c(2L, 1:2))
ii3 <- Rmpfr:::mat2ind(i3, dim(AA), dimnames(AA))
stopifnot(aa[i3] == new("mpfr", getD(AA)[ii3]))
stopifnot(identical(aa[i3], asNumeric(AA[i3])))
CA <- AA; ca <- aa
ca[i3] <- ca[i3] ^ 3
CA[i3] <- CA[i3] ^ 3

## scale():
S2. <- scale(S2)
stopifnot(all.equal(abs(as.vector(S2.)), rep(sqrt(1/mpfr(2, 128)), 4),
  tol = 1e-30))

## norm() :
norm(S2)
stopifnot(identical(norm(S2), norm(S2, "1")),
  norm(S2, "1") == 6,
  norm(S2, "M") == 4,
  abs(norm(S2, "F") - 5.477225575051661) < 1e-15)

```

---

## Description

`determinant(x, ...)` computes the determinant of the mpfr square matrix `x`. May work via coercion to "numeric", i.e., compute `determinant(asNumeric(x), logarithm)`, if `asNumeric` is true, by default, if the dimension is larger than three. Otherwise, use precision `precBits` for the "accumulator" of the result, and use the recursive mathematical definition of the determinant (with computational complexity  $n!$ , where  $n$  is the matrix dimension, i.e., **very** inefficient for all but small matrices!)

## Usage

```

## S3 method for class 'mpfrMatrix'
determinant(x, logarithm = TRUE,
  asNumeric = (d[1] > 3), precBits = max(.getPrec(x)), ...)

```

**Arguments**

<code>x</code>	an <code>mpfrMatrix</code> object of <i>square</i> dimension.
<code>logarithm</code>	logical indicating if the <code>log</code> of the absolute determinant should be returned.
<code>asNumeric</code>	logical ... if rather <code>determinant(asNumeric(x), ...)</code> should be computed.
<code>precBits</code>	the number of binary digits for the result (and the intermediate accumulations).
<code>...</code>	unused (potentially further arguments passed to methods).

**Value**

<code>as determinant()</code>	an object of S3 class "det", a <code>list</code> with components
<code>modulus</code>	the (logarithm of) the absolute value ( <code>abs</code> ) of the determinant of <code>x</code> .
<code>sign</code>	the sign of the determinant.

**Author(s)**

Martin Maechler

**See Also**

`determinant` in base R, which relies on a fast LU decomposition. `mpfrMatrix`

**Examples**

```
m6 <- mpfrArray(1:6, prec=128, dim = c(2L, 3L))
m6
S2 <- m6[,-3] # 2 x 2
S3 <- rbind(m6, c(1:2,10))
det(S2)
str(determinant(S2))
det(S3)
stopifnot(all.equal(det(S2), det(asNumeric(S2)), tolerance=1e-15),
  all.equal(det(S3), det(asNumeric(S3)), tolerance=1e-15))
```

---

optimizeR

*High Precision One-Dimensional Optimization*

---

**Description**

`optimizeR` searches the interval from `lower` to `upper` for a minimum of the function `f` with respect to its first argument.

**Usage**

```
optimizeR(f, lower, upper, ..., tol = 1e-20,
  method = c("Brent", "GoldenRatio"),
  maximum = FALSE,
  precFactor = 2.0, precBits = -log2(tol) * precFactor,
  maxiter = 1000, trace = FALSE)
```

**Arguments**

f	the function to be optimized. $f(x)$ must work “in <b>Rmpfr</b> arithmetic” for <code>optimizer()</code> to make sense. The function is either minimized or maximized over its first argument depending on the value of <code>maximum</code> .
...	additional named or unnamed arguments to be passed to f.
lower	the lower end point of the interval to be searched.
upper	the upper end point of the interval to be searched.
tol	the desired accuracy, typically higher than double precision, i.e., $\text{tol} < 2e-16$ .
method	<b>character</b> string specifying the optimization method.
maximum	logical indicating if $f()$ should be maximized or minimized (the default).
precFactor	only for default <code>precBits</code> construction: a factor to multiply with the number of bits directly needed for <code>tol</code> .
precBits	number of bits to be used for <code>mpfr</code> numbers used internally.
maxiter	maximal number of iterations to be used.
trace	integer or logical indicating if and how iterations should be monitored; if an integer $k$ , print every $k$ -th iteration.

**Details**

“Brent”: Brent(1973)’s simple and robust algorithm is a hybrid, using a combination of the golden ratio and local quadratic (“parabolic”) interpolation. This is the same algorithm as standard R’s `optimize()`, adapted to high precision numbers.

In smooth cases, the convergence is considerably faster than the golden section or Fibonacci ratio algorithms.

“GoldenRatio”: The golden ratio method, aka ‘golden-section search’ works as follows: from a given interval containing the solution, it constructs the next point in the golden ratio between the interval boundaries.

**Value**

A **list** with components `minimum` (or `maximum`) and `objective` which give the location of the minimum (or maximum) and the value of the function at that point; `iter` specifying the number of iterations, the logical `convergence` indicating if the iterations converged and `estim.prec` which is an estimate or an upper bound of the final precision (in  $x$ ). `method` the string of the method used.

**Author(s)**

“GoldenRatio” is based on Hans Werner Borchers’ `golden_ratio` (package **pracma**); modifications and “Brent” by Martin Maechler.

**See Also**

R’s standard `optimize`; for multivariate optimization, **Rmpfr**’s `hjkMpfr()`; for root finding, **Rmpfr**’s `unirootR`.

## Examples

```

## The minimum of the Gamma (and lgamma) function (for x > 0):
Gmin <- optimizeR(gamma, .1, 3, tol = 1e-50)
str(Gmin, digits = 8)
## high precision chosen for "objective"; minimum has "estim.prec" = 1.79e-50
Gmin[c("minimum", "objective")]
## it is however more accurate to 59 digits:
asNumeric(optimizeR(gamma, 1, 2, tol = 1e-100)$minimum - Gmin$minimum)

iG5 <- function(x) -exp(-(x-5)^2/2)
curve(iG5, 0, 10, 200)
o.dp <- optimize(iG5, c(0, 10)) #-> 5 of course
oM.gs <- optimizeR(iG5, 0, 10, method="Golden")
oM.Br <- optimizeR(iG5, 0, 10, method="Brent", trace=TRUE)
oM.gs$min ; oM.gs$iter
oM.Br$min ; oM.Br$iter
(doExtras <- Rmpfr::doExtras())
if(doExtras) {## more accuracy {takes a few seconds}
  oM.gs <- optimizeR(iG5, 0, 10, method="Golden", tol = 1e-70)
  oM.Br <- optimizeR(iG5, 0, 10, tol = 1e-70)
}
rbind(Golden = c(err = as.numeric(oM.gs$min - 5), iter = oM.gs$iter),
      Brent = c(err = as.numeric(oM.Br$min - 5), iter = oM.Br$iter))

## ==> Brent is orders of magnitude more efficient !

## Testing on the sine curve with 40 correct digits:
sol <- optimizeR(sin, 2, 6, tol = 1e-40)
str(sol)
sol <- optimizeR(sin, 2, 6, tol = 1e-50,
                 precFactor = 3.0, trace = TRUE)
pi.. <- 2*sol$min/3
print(pi.., digits=51)
stopifnot(all.equal(pi.., Const("pi", 256), tolerance = 10*1e-50))

if(doExtras) { # considerably more expensive

## a harder one:
f.sq <- function(x) sin(x-2)^4 + sqrt(pmax(0, (x-1)*(x-4)))*(x-2)^2
curve(f.sq, 0, 4.5, n=1000)
msq <- optimizeR(f.sq, 0, 5, tol = 1e-50, trace=5)
str(msq) # ok
stopifnot(abs(msq$minimum - 2) < 1e-49)

## find the other local minimum: -- non-smooth ==> Golden ratio -section is used
msq2 <- optimizeR(f.sq, 3.5, 5, tol = 1e-50, trace=10)
stopifnot(abs(msq2$minimum - 4) < 1e-49)

## and a local maximum:
msq3 <- optimizeR(f.sq, 3, 4, maximum=TRUE, trace=2)
stopifnot(abs(msq3$maximum - 3.57) < 1e-2)

```

```

}#end {doExtras}

##----- "impossible" one to get precisely -----

ff <- function(x) exp(-1/(x-8)^2)
curve(exp(-1/(x-8)^2), -3, 13, n=1001)
(opt. <- optimizeR(function(x) exp(-1/(x-8)^2), -3, 13, trace = 5))
## -> close to 8 {but not very close!}
ff(opt.$minimum) # gives 0
if(doExtras) {
  ## try harder ... in vain ..
  str(opt1 <- optimizeR(ff, -3,13, tol = 1e-60, precFactor = 4))
  print(opt1$minimum, digits=20)
  ## still just 7.99998038 or 8.000036655 {depending on method}
}

```

---

pbetaI

*Accurate Incomplete Beta / Beta Probabilities For Integer Shapes*


---

## Description

For integers  $a, b$ ,  $I_x(a, b)$  aka `pbeta(x, a, b)` is a polynomial in  $x$  with rational coefficients, and hence arbitrarily accurately computable.

TODO (*not yet*): It's sufficient for *one* of  $a, b$  to be integer such that the result is a *finite sum* (but the coefficients will no longer be rational, see Abramowitz and Stegun, 26.5.6 and \*.7, p.944).

## Usage

```

pbetaI(q, shape1, shape2, ncp = 0, lower.tail = TRUE, log.p = FALSE,
       precBits = NULL,
       useRational = !log.p && !is.mpfr(q) && is.null(precBits) && int2,
       rnd.mode = c("N", "D", "U", "Z", "A"))

```

## Arguments

<code>q</code>	called $x$ , above; vector of quantiles, in $[0, 1]$ ; can be <code>numeric</code> , or of class <code>"mpfr"</code> or also <code>"bigq"</code> ("big rational" from package <code>gmp</code> ); in the latter case, if <code>log.p = FALSE</code> as by default, <i>all computations</i> are exact, using big rational arithmetic.
<code>shape1, shape2</code>	the positive Beta "shape" parameters, called $a, b$ , above. <b>Must</b> be integer valued for this function.
<code>ncp</code>	unused, only for compatibility with <code>pbeta</code> , must be kept at its default, 0.
<code>lower.tail</code>	logical; if TRUE (default), probabilities are $P[X \leq x]$ , otherwise, $P[X > x]$ .
<code>log.p</code>	logical; if TRUE, probabilities $p$ are given as $\log(p)$ .
<code>precBits</code>	the precision (in number of bits) to be used in <code>sumBinomMpfr()</code> .

useRational      optional `logical`, specifying if we should try to do everything in exact *rational arithmetic*, i.e., using package `gmp` functionality only, and return `bigq` numbers instead of `mpfr` numbers.

rnd.mode        a 1-letter string specifying how *rounding* should happen at C-level conversion to MPFR, see `mpfr`.

### Value

an `"mpfr"` vector of the same length as `q`.

### Note

For upper tail probabilities, i.e., when `lower.tail=FALSE`, we may need large `precBits`, because the implicit or explicit  $1 - P$  computation suffers from severe cancellation.

### Author(s)

Martin Maechler

### References

Abramowitz, M. and Stegun, I. A. (1972) *Handbook of Mathematical Functions*. New York: Dover. [https://en.wikipedia.org/wiki/Abramowitz\\_and\\_Stegun](https://en.wikipedia.org/wiki/Abramowitz_and_Stegun) provides links to the full text which is in public domain.

### See Also

`pbeta`, `sumBinomMpfr` `chooseZ`.

### Examples

```
x <- (0:12)/16 # not all the way up ..
a <- 7; b <- 788

p. <- pbetaI(x, a, b) ## a bit slower:
system.time(
pp <- pbetaI(x, a, b, precBits = 2048)
) # 0.23 -- 0.50 sec
## Currently, the lower.tail=FALSE are computed "badly":
lp <- log(pp)      ## = pbetaI(x, a, b, log.p=TRUE)
llp <- log1p(-pp) ## = pbetaI(x, a, b, lower.tail=FALSE, log.p=TRUE)
lp <- 1 - pp      ## = pbetaI(x, a, b, lower.tail=FALSE)

if(Rmpfr:::doExtras()) { ## somewhat slow
  stopifnot(
    all.equal(lp, pbetaI(x, a, b, precBits = 2048, log.p=TRUE)),
    all.equal(llp, pbetaI(x, a, b, precBits = 2048, lower.tail=FALSE, log.p=TRUE),
              tol = 1e-230),
    all.equal(lp, pbetaI(x, a, b, precBits = 2048, lower.tail=FALSE))
  )
}
```

```

rErr <- function(approx, true, eps = 1e-200) {
  true <- as.numeric(true) # for "mpfr"
  ifelse(Mod(true) >= eps,
        ## relative error, catching '-Inf' etc :
  ifelse(true == approx, 0, 1 - approx / true),
        ## else: absolute error (e.g. when true=0)
  true - approx)
}

rErr(pbeta(x, a, b), pp)
rErr(pbeta(x, a, b, lower=FALSE), Ip)
rErr(pbeta(x, a, b, log = TRUE), lp)
rErr(pbeta(x, a, b, lower=FALSE, log = TRUE), lIp)

a.EQ <- function(..., tol=1e-15) all.equal(..., tolerance=tol)
stopifnot(
  a.EQ(pp, pbeta(x, a, b)),
  a.EQ(lp, pbeta(x, a, b, log.p=TRUE)),
  a.EQ(lIp, pbeta(x, a, b, lower.tail=FALSE, log.p=TRUE)),
  a.EQ(Ip, pbeta(x, a, b, lower.tail=FALSE))
)

## When 'q' is a bigrational (i.e., class "bigq", package 'gmp'), everything
## is computed *exactly* with bigrational arithmetic:
(q4 <- as.bigq(1, 2^(0:4)))
pb4 <- pbetaI(q4, 10, 288, lower.tail=FALSE)
stopifnot( is.bigq(pb4) )
mpb4 <- as(pb4, "mpfr")
mpb4[1:2]
getPrec(mpb4) # 128 349 1100 1746 2362
(pb. <- pbeta(asNumeric(q4), 10, 288, lower.tail=FALSE))
stopifnot(mpb4[1] == 0,
          all.equal(mpb4, pb., tol=4e-15))

qbetaI. <- function(p, shape1, shape2, ncp = 0, lower.tail = TRUE, log.p = FALSE,
  precBits = NULL, rnd.mode = c("N", "D", "U", "Z", "A"),
  tolerance = 1e-20, ...)
{
  if(is.na(a <- as.integer(shape1))) stop("a = shape1 is not coercable to finite integer")
  if(is.na(b <- as.integer(shape2))) stop("b = shape2 is not coercable to finite integer")
  unirootR(function(q) pbetaI(q, a, b, lower.tail=lower.tail, log.p=log.p,
    precBits=precBits, rnd.mode=rnd.mode) - p,
    interval = if(log.p) c(-double.xmax, 0) else 0:1,
    tol = tolerance, ...)
} # end{qbetaI}

(p <- 1 - mpfr(1,128)/20) # 'p' must be high precision
q95.1.3 <- qbetaI.(p, 1,3, tolerance = 1e-29) # -> ~29 digits accuracy
str(q95.1.3) ; roundMpfr(q95.1.3$root, precBits = 29 * log2(10))
## relative error is really small:
(relE <- asNumeric(1 - pbetaI(q95.1.3$root, 1,3) / p))
stopifnot(abs(relE) < 1e-28)

```

---

pmax

*Parallel Maxima and Minima*

---

### Description

Returns the parallel maxima and minima of the input values.

The functions `pmin` and `pmax` have been made S4 generics, and this page documents the “... method for class `"mNumber"`”, i.e., for arguments that are numeric or from `class "mpfr"`.

### Usage

```
pmax(..., na.rm = FALSE)
pmin(..., na.rm = FALSE)
```

### Arguments

`...` numeric or arbitrary precision numbers (class `mpfr`).

`na.rm` a logical indicating whether missing values should be removed.

### Details

See [pmax](#), the documentation of the base functions, i.e., default methods.

### Value

vector-like, of length the longest of the input vectors; typically of class `mpfr`, for the methods here.

### Methods

`...` = `"ANY"` the default method, really just `base::pmin` or `base::pmax`, respectively.

`...` = `"mNumber"` the method for `mpfr` arguments, mixed with numbers; designed to follow the same semantic as the default method.

### See Also

The documentation of the **base** functions, [pmin](#) and [pmax](#); also [min](#) and [max](#); further, [range](#) (*both* `min` and `max`).

### Examples

```
(pm <- pmin(1.35, mpfr(0:10, 77)))
stopifnot(pm == pmin(1.35, 0:10))
```



---

Rmpfr-workarounds      *Base Functions etc, as an Rmpfr version*

---

**Description**

Functions from **base** etc which need a *copy* in the **Rmpfr** namespace so they correctly dispatch.

**Usage**

```
outer(X, Y, FUN = "*", ...)
```

**Arguments**

X, Y, FUN, ...    See **base** package help: [outer](#).

**See Also**

[outer](#).

**Examples**

```
outer(1/mpfr(1:10, 70), 0:2)
```

---

roundMpfr      *Rounding to Binary bits, "mpfr-internally"*

---

**Description**

Rounding to binary bits, not decimal digits. Closer to the number representation, this also allows to *increase* or decrease a number's `precBits`. In other words, it acts as `setPrec()`, see [getPrec\(\)](#).

**Usage**

```
roundMpfr(x, precBits, rnd.mode = c("N", "D", "U", "Z", "A"))
```

**Arguments**

x	an mpfr number (vector)
precBits	integer specifying the desired precision in bits.
rnd.mode	a 1-letter string specifying how <i>rounding</i> should happen at C-level conversion to MPFR, see <a href="#">mpfr</a> .

**Value**

an mpfr number as x but with the new 'precBits' precision

**See Also**

The `mpfr` class group method `Math2` implements a method for `round(x, digits)` which rounds to *decimal* digits.

**Examples**

```
(p1 <- Const("pi", 100)) # 100 bit prec
roundMpfr(p1, 120) # 20 bits more, but "random noise"
Const("pi", 120) # same "precision", but really precise
```

---

sapplyMpfr

*Apply a Function over a "mpfr" Vector*


---

**Description**

Users may be disappointed to note that `sapply()` or `vapply()` typically do not work with "mpfr" numbers.

This is a simple (but strong) approach to work around the problem, based on `lapply()`.

Note that this is not yet as flexible as `sapply()` for atomic vectors.

**Usage**

```
sapplyMpfr(X, FUN, ...)
```

**Arguments**

X	a vector, possibly of class "mpfr".
FUN	a function returning an "mpfr" number. (TODO: A function returning a <i>vector</i> of mpfr numbers or even "mpfrArray").
...	further arguments passed to <code>lapply</code> , typically further arguments to FUN.

**Value**

an "mpfr" vector, typically of the same length as X.

**Note**

Another workaround could be to use

```
res <- lapply(...)
sapply(res, asNumeric)
```

**Author(s)**

Martin Maechler

**Examples**

```
## The function is simply defined as
function (X, FUN, ...)
  new("mpfr", unlist(lapply(X, FUN, ...), recursive = FALSE))

if(require("Bessel")) # here X, is simple
  bImp <- sapplyMpfr(0:4, function(k)
    besseli.nuAsym(mpfr(1.31e9, 128), 10, expon.scaled=TRUE, k.max=k))
```

seqMpfr

*"mpfr" Sequence Generation***Description**

Generate 'regular', i.e., arithmetic sequences. This is in lieu of methods for [seq](#) (dispatching on all three of from, to, and by).

**Usage**

```
seqMpfr(from = 1, to = 1, by = ((to - from)/(length.out - 1)),
        length.out = NULL, along.with = NULL, ...)
```

**Arguments**

from, to	the starting and (maximal) end value (numeric or "mpfr") of the sequence.
by	number (numeric or "mpfr"): increment of the sequence.
length.out	desired length of the sequence. A non-negative number, which will be rounded up if fractional.
along.with	take the length from the length of this argument.
...	arguments passed to or from methods.

**Details**

see [seq](#) (default method in package **base**), whose semantic we want to replicate (almost).

**Value**

a 'vector' of class "mpfr", when one of the first three arguments was.

**Author(s)**

Martin Maechler

**See Also**

The documentation of the **base** function [seq](#); [mpfr](#)

**Examples**

```
seqMpfr(0, 1, by = mpfr(0.25, prec=88))
```

```
seqMpfr(7, 3) # -> default prec.
```

---

```
str.mpfr
```

---

*Compactly Show STRucture of Rmpfr Number Object*

---

**Description**

The `str` method for objects of class `mpfr` produces a bit more useful output than the default method `str.default`.

**Usage**

```
## S3 method for class 'mpfr'
str(object, nest.lev, internal = FALSE,
     give.head = TRUE, digits.d = 12, vec.len = NULL, drop0trailing=TRUE,
     width = getOption("width"), ...)
```

**Arguments**

<code>object</code>	an object of class <code>mpfr</code> .
<code>nest.lev</code>	for <code>str()</code> , typically only used when called by a higher level <code>str()</code> .
<code>internal</code>	logical indicating if the low-level internal structure should be shown; if true (not by default), uses <code>str(object@.Data)</code> .
<code>give.head</code>	logical indicating if the “header” should be printed.
<code>digits.d</code>	the number of digits to be used, will be passed <code>formatMpfr()</code> and hence NULL will use “as many as needed”, i.e. often too many. If this is a number, as per default, less digits will be used in case the precision ( <code>getPrec(object)</code> ) is smaller.
<code>vec.len</code>	the number of <i>elements</i> that will be shown. The default depends on the precision of object and width (since <b>Rmpfr</b> 0.6-0, it was 3 previously).
<code>drop0trailing</code>	logical, passed to <code>formatMpfr()</code> (with a different default here).
<code>width</code>	the (approximately) desired width of output, see <code>options(width = .)</code> .
<code>...</code>	further arguments, passed to <code>formatMpfr()</code> .

**See Also**

`.mpfr2list()` puts the internal structure into a `list`, and its help page documents many more (low level) utilities.

**Examples**

```
(x <- c(Const("pi", 64), mpfr(-2:2, 64)))
str(x)
str(list(pi = pi, x.mpfr = x))
str(x ^ 1000)
str(x ^ -1e4, digits=NULL) # full precision

str(x, internal = TRUE) # internal low-level (for experts)

uu <- Const("pi", 16) # unaccurate
str(uu) # very similar to just 'uu'
```

sumBinomMpfr

*(Alternating) Binomial Sums via Rmpfr***Description**

Compute (alternating) binomial sums via high-precision arithmetic. If  $sBn(f, n) := \text{sumBinomMpfr}(n, f)$ , (default alternating is true, and  $n_0 = 0$ ),

$$sBn(f, n) = \sum_{k=n_0}^n (-1)^{(n-k)} \binom{n}{k} \cdot f(k) = \Delta^n f,$$

see Details for the  $n$ -th forward difference operator  $\Delta^n f$ . If alternating is false, the  $(-1)^{(n-k)}$  factor is dropped (or replaced by 1) above.

Such sums appear in different contexts and are typically challenging, i.e., currently impossible, to evaluate reliably as soon as  $n$  is larger than around 50 – 70.

**Usage**

```
sumBinomMpfr(n, f, n0 = 0, alternating = TRUE, precBits = 256,
             f.k = f(mpfr(k, precBits=precBits)))
```

**Arguments**

n	upper summation index (integer).
f	<b>function</b> to be evaluated at $k$ for $k$ in $n_0:n$ (and which must return <i>one</i> value per $k$ ).
n0	lower summation index, typically 0 (= default) or 1.
alternating	logical indicating if the sum is alternating, see below.
precBits	the number of bits for MPFR precision, see <a href="#">mpfr</a> .
f.k	can be specified instead of $f$ and $\text{precBits}$ , and must contain the equivalent of its default, $f(\text{mpfr}(k, \text{precBits}=\text{precBits}))$ .

**Details**

The alternating binomial sum  $sB(f, n) := \text{sumBinom}(n, f, n0 = 0)$  is equal to the  $n$ -th forward difference operator  $\Delta^n f$ ,

$$sB(f, n) = \Delta^n f,$$

where

$$\Delta^n f = \sum_{k=0}^n (-1)^{n-k} \binom{n}{k} \cdot f(k),$$

is the  $n$ -fold iterated forward difference  $\Delta f(x) = f(x+1) - f(x)$  (for  $x = 0$ ).

The current implementation might be improved in the future, notably for the case where  $sB(f, n) = \text{sumBinomMpfR}(n, f, *)$  is to be computed for a whole sequence  $n = 1, \dots, N$ .

**Value**

an `mpfr` number of precision `precBits`. `s`. If `alternating` is true (as per default),

$$s = \sum_{k=n0}^n (-1)^k \binom{n}{k} \cdot f(k),$$

if `alternating` is false, the  $(-1)^k$  factor is dropped (or replaced by 1) above.

**Author(s)**

Martin Maechler, after conversations with Christophe Dutang.

**References**

Wikipedia (2012) The N"orlund-Rice integral, [https://en.wikipedia.org/wiki/Rice\\_integral](https://en.wikipedia.org/wiki/Rice_integral)  
 Flajolet, P. and Sedgewick, R. (1995) Mellin Transforms and Asymptotics: Finite Differences and Rice's Integrals, *Theoretical Computer Science* **144**, 101–124.

**See Also**

[chooseMpfR](#), [chooseZ](#) from package `gmp`.

**Examples**

```
## "naive" R implementation:
sumBinom <- function(n, f, n0=0, ...) {
  k <- n0:n
  sum( choose(n, k) * (-1)^(n-k) * f(k, ...) )
}

## compute sumBinomMpfR(.) for a whole set of 'n' values:
sumBin.all <- function(n, f, n0=0, precBits = 256, ...)
{
  N <- length(n)
  precBits <- rep(precBits, length = N)
  ll <- lapply(seq_len(N), function(i)
```

```

        sumBinomMpfr(n[i], f, n0=n0, precBits=precBits[i], ...)
    sapply(1l, as, "double")
}
sumBin.all.R <- function(n, f, n0=0, ...)
  sapply(n, sumBinom, f=f, n0=n0, ...)

n.set <- 5:80
system.time(res.R <- sumBin.all.R(n.set, f = sqrt)) ## instantaneous..
system.time(resMpfr <- sumBin.all (n.set, f = sqrt)) ## ~ 0.6 seconds

matplot(n.set, cbind(res.R, resMpfr), type = "l", lty=1,
        ylim = extendrange(resMpfr, f = 0.25), xlab = "n",
        main = "sumBinomMpfr(n, f = sqrt) vs. R double precision")
legend("topleft", leg=c("double prec.", "mpfr"), lty=1, col=1:2, bty = "n")

```

unirootR

*One Dimensional Root (Zero) Finding – in pure R***Description**

The function `unirootR` searches the interval from `lower` to `upper` for a root (i.e., zero) of the function `f` with respect to its first argument.

`unirootR()` is “clone” of `uniroot()`, written entirely in R, in a way that it works with `mpfr`-numbers as well.

**Usage**

```

unirootR(f, interval, ...,
        lower = min(interval), upper = max(interval),
        f.lower = f(lower, ...), f.upper = f(upper, ...),
        verbose = FALSE,
        tol = .Machine$double.eps^0.25, maxiter = 1000,
        warn.no.convergence = TRUE,
        epsC = NULL)

```

**Arguments**

<code>f</code>	the function for which the root is sought.
<code>interval</code>	a vector containing the end-points of the interval to be searched for the root.
<code>...</code>	additional named or unnamed arguments to be passed to <code>f</code>
<code>lower, upper</code>	the lower and upper end points of the interval to be searched.
<code>f.lower, f.upper</code>	the same as <code>f(upper)</code> and <code>f(lower)</code> , respectively. Passing these values from the caller where they are often known is more economical as soon as <code>f()</code> contains non-trivial computations.
<code>verbose</code>	logical (or integer) indicating if (and how much) verbose output should be produced during the iterations.

tol	the desired accuracy (convergence tolerance).
maxiter	the maximum number of iterations.
warn.no.convergence	if set to FALSE there's no warning about non-convergence. Useful to just run a few iterations.
epsC	positive number or NULL in which case a smart default is sought. This should specify the “achievable machine precision” <i>for</i> the given numbers and their arithmetic. The default will set this to <code>.Machine\$double.eps</code> for double precision numbers, and will basically use $2^{-\min(\text{getPrec}(f.\text{lower}), \text{getPrec}(f.\text{upper}))}$ when that works (as, e.g., for <code>mpfr</code> -numbers) otherwise. This is factually a lower bound for the achievable lower bound, and hence, setting <code>tol</code> smaller than <code>epsC</code> is typically non-sensical sense and produces a warning.

### Details

Note that arguments after `...` must be matched exactly.

Either `interval` or both `lower` and `upper` must be specified: the upper endpoint must be strictly larger than the lower endpoint. The function values at the endpoints must be of opposite signs (or zero).

The function only uses R code with basic arithmetic, such that it should also work with “generalized” numbers (such as `mpfr`-numbers) as long the necessary `Ops` methods are defined for those.

The underlying algorithm assumes a continuous function (which then is known to have at least one root in the interval).

Convergence is declared either if  $f(x) == 0$  or the change in  $x$  for one step of the algorithm is less than `tol` (plus an allowance for representation error in  $x$ ).

If the algorithm does not converge in `maxiter` steps, a warning is printed and the current approximation is returned.

`f` will be called as `f(x, ...)` for a (generalized) numeric value of  $x$ .

### Value

A list with four components: `root` and `f.root` give the location of the root and the value of the function evaluated at that point. `iter` and `estim.prec` give the number of iterations used and an approximate estimated precision for root. (If the root occurs at one of the endpoints, the estimated precision is NA.)

### Source

Based on `zeroin()` (in package **rootoned**) by John Nash who manually translated the C code in R's `zeroin.c` and on `uniroot()` in R's sources.

### References

Brent, R. (1973), see `uniroot`.



**See Also**

[polyroot](#) for all complex roots of a polynomial; [optimize](#), [nlm](#).

**Examples**

```
require(utils) # for str

## some platforms hit zero exactly on the first step:
## if so the estimated precision is 2/3.
f <- function (x,a) x - a
str(xmin <- unirootR(f, c(0, 1), tol = 0.0001, a = 1/3))

## handheld calculator example: fixpoint of cos(.):
rc <- unirootR(function(x) cos(x) - x, lower=-pi, upper=pi, tol = 1e-9)
rc$root

## the same with much higher precision:
rcM <- unirootR(function(x) cos(x) - x,
                interval= mpfr(c(-3,3), 300), tol = 1e-40)
rcM
x0 <- rcM$root
stopifnot(all.equal(cos(x0), x0,
                    tol = 1e-40))## 40 digits accurate!

str(unirootR(function(x) x*(x^2-1) + .5, lower = -2, upper = 2,
            tol = 0.0001), digits.d = 10)
str(unirootR(function(x) x*(x^2-1) + .5, lower = -2, upper = 2,
            tol = 1e-10 ), digits.d = 10)

## A sign change of f(.), but not a zero but rather a "pole":
tan. <- function(x) tan(x * (Const("pi",200)/180))# == tan( <angle> )
(rtan <- unirootR(tan., interval = mpfr(c(80,100), 200), tol = 1e-40))
## finds 90 {"ok"}, and now gives a warning

## Find the smallest value x for which exp(x) > 0 (numerically):
r <- unirootR(function(x) 1e80*exp(x)-1e-300, c(-1000,0), tol = 1e-15)
str(r, digits.d = 15) ##> around -745, depending on the platform.

exp(r$root)      # = 0, but not for r$root * 0.999...
minexp <- r$root * (1 - 10*.Machine$double.eps)
exp(minexp)      # typically denormalized

## --- using mpfr-numbers :

## Find the smallest value x for which exp(x) > 0 ("numerically");
## Note that mpfr-numbers underflow *MUCH* later than doubles:
## one of the smallest mpfr-numbers {see also ?mpfr-class } :
(ep.M <- mpfr(2, 55) ^ - ((2^30 + 1) * (1 - 1e-15)))
r <- unirootR(function(x) 1e99* exp(x) - ep.M, mpfr(c(-1e20, 0), 200))
r # 97 iterations; f.root is very similar to ep.M
```

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