

# Package ‘NPCirc’

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## Description

Nonparametric smoothing methods for density and regression estimation involving circular data, including methods described in Oliveira et al. (2014) and proposals in Alonso-Pena et al. (2021).

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NPCirc-package . . . . .	2
ancova.circ.lin . . . . .	4
bw.boot . . . . .	6
bw.CV . . . . .	8
bw.pi . . . . .	9

bw.reg.circ.lin . . . . .	10
bw.rt . . . . .	12
circsizer.density . . . . .	13
circsizer.map . . . . .	16
circsizer.regression . . . . .	17
cross.beds1 . . . . .	19
cross.beds2 . . . . .	19
cycle.changes . . . . .	20
dcirmix . . . . .	21
dragonfly . . . . .	23
dwsn . . . . .	24
kern.den.circ . . . . .	25
kern.reg.circ.lin . . . . .	27
lines.regression.circular . . . . .	29
noeffect.circ.lin . . . . .	31
periwinkles . . . . .	33
plot.regression.circular . . . . .	34
speed.wind . . . . .	35
temp.wind . . . . .	36
wind . . . . .	37

**Index** **39**

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NPCirc-package	<i>Nonparametric circular methods.</i>
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**Description**

Nonparametric smoothing methods for density and regression estimation involving circular data, including methods described in Oliveira et al. (2014) and proposals in Alonso-Pena et al. (2021).

**Details**

Package:	NPCirc
Type:	Package
Version:	3.0.1
Date:	2021-07-21
License:	GPL-2
LazyLoad:	yes

This package incorporates the function `kern.den.circ` which computes the circular kernel density estimator. For choosing the smoothing parameter different functions are available: `bw.rt`, `bw.CV`, `bw.pi`, and `bw.boot`. For regression involving circular variables, the package includes the functions: `kern.reg.circ.lin` for a circular covariate and linear response; `kern.reg.circ.circ` for a circular covariate and a circular response; `kern.reg.lin.circ` for a linear covariate and a circular response. The three functions compute Nadaraya-Watson and Local-Linear smoothers. The func-

tions `bw.reg.circ.lin`, `bw.reg.circ.circ` and `bw.reg.circ.lin` implement cross-validation rules for selecting the smoothing parameter. Functions `noeffect.circ.lin`, `noeffect.circ.circ` and `noeffect.lin.circ` compute the test of no effect to assess the significance of the predictor variable. Additionally, functions `ancova.circ.lin`, `ancova.circ.circ` and `ancova.lin.circ` implement hypothesis testing tools to assess the equality and parallelism of regression curves across different groups of observations. Functions `circsizer.density` and `circsizer.regression` provide CircSiZer maps for kernel density estimation and regression estimation, respectively. Functions `dcircmix` and `rcircmix` compute the density function and generate random samples of a circular distribution or a mixture of circular distributions, allowing for different components such as the circular uniform, von Mises, cardioid, wrapped Cauchy, wrapped normal and wrapped skew-normal. Finally, some data sets are provided. Missing data are allowed. Registries with missing data are simply removed.

For a complete list of functions, use `library(help="NPCirc")`.

### Acknowledgements

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### Author(s)

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### References

- Oliveira, M., Crujeiras, R.M. and Rodr?guez-Casal, A. (2012) A plug-in rule for bandwidth selection in circular density. *Computational Statistics and Data Analysis*, **56**, 3898–3908.
- Oliveira, M., Crujeiras R.M. and Rodr?guez-Casal, A. (2013) Nonparametric circular methods for exploring environmental data. *Environmental and Ecological Statistics*, **20**, 1–17.
- Oliveira, M., Crujeiras, R.M. and Rodr?guez-Casal (2014) CircSiZer: an exploratory tool for circular data. *Environmental and Ecological Statistics*, **21**, 143–159.
- Oliveira, M., Crujeiras R.M. and Rodr?guez-Casal, A. (2014) NPCirc: an R package for nonparametric circular methods. *Journal of Statistical Software*, **61**(9), 1–26. <https://www.jstatsoft.org/v61/i09/>
- Alonso-Pena, M., Ameijeiras-Alonso, J. and Crujeiras, R.M. (2021) Nonparametric tests for circular regression. *Journal of Statistical Computation and Simulation*, **91**(3), 477–500.

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 ancova.circ.lin

*Nonparametric analysis of covariance tests for circular regression*


---

## Description

Function `ancova.circ.lin` computes nonparametric ANCOVA tests to compare regression curves with a circular predictor variable and a real-valued response variable. The null hypothesis may be either equality or parallelism of the regression curves, as described in Alonso-Pena et al. (2021). It uses the nonparametric Nadaraya-Watson estimator or the Local-Linear estimator for circular-linear data described in Di Marzio et al. (2009) and Oliveira et al. (2013).

Function `ancova.lin.circ` computes nonparametric ANCOVA tests to compare regression curves with a real-valued predictor variable and a circular response variable. The null hypothesis may be either equality or parallelism of the regression curves, as described in Alonso-Pena et al. (2021). It uses the nonparametric Nadaraya-Watson estimator or the Local-Linear estimator for linear-circular data described in Di Marzio et al. (2012).

Function `ancova.circ.circ` computes nonparametric ANCOVA tests to compare regression curves with a circular predictor variable and a circular response variable. The null hypothesis may be either equality or parallelism of the regression curves, as described in Alonso-Pena et al. (2021). It uses the nonparametric Nadaraya-Watson estimator or the Local-Linear estimator for circular-circular data described in Di Marzio et al. (2012).

## Usage

```
ancova.circ.lin(x, y, g, bw, bw1, test = "eq", method = "LL",
               calib = "chisq", n_boot = 500)
ancova.lin.circ(x, y, g, bw, bw1, test = "eq", method = "LL", n_boot = 500)
ancova.circ.circ(x, y, g, bw, bw1, test = "eq", method = "LL", n_boot = 500)
```

## Arguments

<code>x</code>	Vector of data for the independent variable. The object is coerced to class <code>circular</code> when using functions <code>noeffect.circ.lin</code> and <code>noeffect.circ.circ</code> .
<code>y</code>	Vector of data for the dependent variable. This must be same length as <code>x</code> . The object is coerced to class <code>circular</code> when using functions <code>noeffect.lin.circ</code> and <code>noeffect.circ.circ</code> .
<code>g</code>	Vector of group indicators.
<code>bw</code>	Smoothing parameter to be used. If not provided it selects the parameter obtained by cross-validation.
<code>bw1</code>	Preliminary smoothing parameter for the parallelism test.
<code>test</code>	Character string giving the type of test to be performed. Must be one of "eq" for the test of equality or "paral" for the test of parallelism. Default is <code>test="eq"</code> .
<code>method</code>	Character string giving the estimator to be used. This must be one of "LL" for Local-Linear estimator or "NW" for Nadaraya-Watson estimator. Default <code>method="LL"</code> .

calib	Character string giving the calibration method to be used in <code>noeffect.circ.lin</code> function. This must be one of "chisq" for the chi-squared approximation or "boot" for the bootstrap calibration.
n_boot	Number of bootstrap resamples. Default is <code>n_boot=500</code> . In function <code>noeffect.circ.lin</code> , only if <code>calib="boot"</code> .

### Details

See Alonso-Pena et al. (2021). The NAs will be automatically removed.

### Value

A list with class "htest" containing the following components:

statistic	observed value of the statistic.
bw	Smoothing parameter used.
p.value	p-value for the test.
data.name	a character string giving the name(s) of the data.
alternative	a character string describing the alternative hypothesis.

### Author(s)

María Alonso-Pena, Jose Ameijeiras-Alonso and Rosa M. Crujeiras

### References

- Alonso-Pena, M., Ameijeiras-Alonso, J. and Crujeiras, R.M. (2021) Nonparametric tests for circular regression. *Journal of Statistical Computation and Simulation*, **91**(3), 477–500.
- Di Marzio, M., Panzera A. and Taylor, C. C. (2009) Local polynomial regression for circular predictors. *Statistics and Probability Letters*, **79**, 2066–2075.
- Di Marzio, M., Panzera A. and Taylor, C. C. (2012) Non-parametric regression for circular responses. *Scandinavian Journal of Statistics*, **40**, 228–255.
- Oliveira, M., Crujeiras R.M. and Rodríguez-Casal, A. (2013) Nonparametric circular methods for exploring environmental data. *Environmental and Ecological Statistics*, **20**, 1–17.

### See Also

[kern.reg.circ.lin](#), [kern.reg.lin.circ](#), [kern.reg.circ.circ](#)

### Examples

```
# ANCOVA circ-lin
set.seed(2025)
x1 <- rcircularuniform(100)
x2 <- rcircularuniform(100)
x <- c(x1, x2)
y1 <- 2*sin(as.numeric(x1)) + rnorm(100, sd=2)
```

```

y2 <- 4 + 2*sin(as.numeric(x2)) + rnorm(100, sd=2)
y <- c(y1, y2)
g <- c(rep(0,100), rep(1,100))
ancova.circ.lin(x, y, g, test = "eq")
ancova.circ.lin(x, y, g, test = "paral")

# ANCOVA lin-circ
set.seed(2025)
x1 <- runif(100)
x2 <- runif(100)
y1 <- 3*pi*x1^2 + rvonmises(100, mu = 0, kappa = 6)
y2 <- 2*pi/8 + 3*pi*x2^2 + rvonmises(100, mu = 0, kappa = 6)
x <- c(x1, x2)
y <- c(y1, y2)
g<-c(rep(0, 100), rep(1, 100))
ancova.lin.circ(x, y, g, test = "eq")
ancova.lin.circ(x, y, g, test = "paral")

# ANCOVA circ-circ
set.seed(2025)
x1 <- rcircularuniform(100)
x2 <- rcircularuniform(100)
y1 <- 2*sin(2*x1) + rvonmises(100, mu = 0, kappa = 8 )
y2 <- pi/8 + 2*sin(2*x2) + rvonmises(100, mu = 0, kappa = 8 )
x <- c(x1, x2)
y <- c(y1, y2)
g<-c(rep(0, 100), rep(1, 100))
ancova.circ.circ(x, y, g, test = "eq")
ancova.circ.circ(x, y, g, test = "paral")

```

---

 bw.boot

*Bootstrap method*


---

## Description

This function implements the bootstrap procedure proposed by Di Marzio et al. (2011) for selecting the smoothing parameter for density estimation taking the von Mises density as kernel.

## Usage

```
bw.boot(x, lower=0, upper=100, np=500, tol=0.1)
```

**Arguments**

x	Data from which the smoothing parameter is to be computed. The object is coerced to class <code>circular</code> .
lower, upper	lower and upper boundary of the interval to be used in the search for the value of the smoothing parameter. Default lower=0 and upper=100.
np	Number of points where to evaluate the estimator for numerical integration. Default np=500.
tol	Convergence tolerance for <code>optimize</code> .

**Details**

This method is based on the proposal of Taylor (1989) for linear data. See also Oliveira et al. (2012). The NAs will be automatically removed.

**Value**

Value of the smoothing parameter.

**Author(s)**

Mar?a Oliveira, Rosa M. Crujeiras and Alberto Rodr?guez-Casal

**References**

Di Marzio, M., Panzera A. and Taylor, C.C. (2011) Kernel density estimation on the torus. *Journal of Statistical Planning and Inference*, **141**, 2156–2173.

Oliveira, M., Crujeiras, R.M. and Rodr?guez-Casal, A. (2012) A plug-in rule for bandwidth selection in circular density. *Computational Statistics and Data Analysis*, **56**, 3898–3908.

Taylor, C.C. (1989) Bootstrap choice of the smoothing parameter in kernel density estimation. *Biometrika*, **76**, 705–712.

Oliveira, M., Crujeiras R.M. and Rodr?guez-Casal, A. (2014) NPCirc: an R package for nonparametric circular methods. *Journal of Statistical Software*, **61**(9), 1–26. <https://www.jstatsoft.org/v61/i09/>

**See Also**

`kern.den.circ`, `bw.rt`, `bw.CV`, `bw.pi`

**Examples**

```
set.seed(2012)
n <- 100
x <- rcircmix(n, model=17)
bw.boot(x, lower=0, upper=20)
```

bw.CV

*Cross-validation for density estimation***Description**

This function provides a least squares cross-validation smoothing parameter or a likelihood cross-validation smoothing parameter for density estimation.

**Usage**

```
bw.CV(x, method="LCV", lower=0, upper=50, tol=1e-2, np=500)
```

**Arguments**

x	Data from which the smoothing parameter is to be computed. The object is coerced to class <code>circular</code> .
method	Character string giving the cross-validation rule to be used. This must be one of "LCV" or "LSCV". Default method="LCV".
lower, upper	lower and upper boundary of the interval to be used in the search for the value of the smoothing parameter. Default lower=0 and upper=50.
tol	Convergence tolerance for <code>optimize</code> . Default tol=1e-2.
np	Number of points where to evaluate the estimator for numerical integration when method="LSCV". Default np=500.

**Details**

The LCV smoothing parameter is obtained as the value of  $\nu$  that maximizes the logarithm of the likelihood cross-validation function (8) in Oliveira et al. (2013). The LSCV smoothing parameter is obtained as the value of  $\nu$  that minimizes expression (7) in Oliveira et al. (2013). See also Hall et al. (1987) and Oliveira et al. (2012). The NAs will be automatically removed.

**Value**

Value of the smoothing parameter.

**Author(s)**

María Oliveira, Rosa M. Crujeiras and Alberto Rodríguez-Casal

**References**

Hall, P., Watson, G.S. and Cabrera, J. (1987) Kernel density estimation with spherical data, *Biometrika*, **74**, 751–762.

Oliveira, M., Crujeiras, R.M. and Rodríguez-Casal, A. (2012) A plug-in rule for bandwidth selection in circular density. *Computational Statistics and Data Analysis*, **56**, 3898–3908.



Oliveira, M., Crujeiras R.M. and Rodríguez-Casal, A. (2013) Nonparametric circular methods for exploring environmental data. *Environmental and Ecological Statistics*, **20**, 1–17.

Oliveira, M., Crujeiras R.M. and Rodríguez-Casal, A. (2014) NPCirc: an R package for nonparametric circular methods. *Journal of Statistical Software*, **61**(9), 1–26. <https://www.jstatsoft.org/v61/i09/>

### See Also

[kern.den.circ](#), [bw.rt](#), [bw.pi](#), [bw.boot](#)

### Examples

```
set.seed(2012)
n <- 100
x <- rcircmix(n, model=11)
bw.CV(x, method="LCV", lower=0, upper=20)
bw.CV(x, method="LSCV", lower=0, upper=20)
```

---

bw.pi

*Plug-in rule*

---

### Description

This function implements the von Mises scale plug-in rule for the smoothing parameter for density estimation when the number of components in the mixture is selected by Akaike Information Criterion (AIC) which selects the best model between a mixture of 2-5 von Mises distributions.

### Usage

```
bw.pi(x, M=NULL, lower=0, upper=100, np=500, tol=0.1, outM=FALSE)
```

### Arguments

x	Data from which the smoothing parameter is to be computed. The object is coerced to class <code>circular</code> .
M	Integer indicating the number of components in the mixture. If M=1, the rule of thumb is carried out with $\kappa$ estimated by maximum likelihood. If M=NULL, AIC will be used.
lower, upper	lower and upper boundary of the interval to be used in the search for the value of the smoothing parameter. Default lower=0 and upper=100.
np	Number of points where to evaluate the estimator for numerical integration. Default np=500.
tol	Convergence tolerance for <code>optimize</code> . Default tol=0.1.
outM	Logical; if TRUE the function also returns the number of components in the mixture. Default, outM=FALSE.

### Details

The value of the smoothing parameter is chosen by minimizing the asymptotic mean integrated squared error (AMISE) derived by Di Marzio et al. (2009) assuming that the data follow a mixture of von Mises distributions. The number of components in the mixture can be fixed by the user, by specifying the argument `M` or selected by using AIC (default option) as described in Oliveira et al. (2012). The NAs will be automatically removed.

### Value

Vector with the value of the smoothing parameter and the number of components in the mixture (if specified).

### Author(s)

Mar?a Oliveira, Rosa M. Crujeiras and Alberto Rodr?guez-Casal

### References

Oliveira, M., Crujeiras, R.M. and Rodr?guez-Casal, A. (2012) A plug-in rule for bandwidth selection in circular density. *Computational Statistics and Data Analysis*, **56**, 3898–3908.

Oliveira, M., Crujeiras R.M. and Rodr?guez-Casal, A. (2014) NPCirc: an R package for nonparametric circular methods. *Journal of Statistical Software*, **61**(9), 1–26. <https://www.jstatsoft.org/v61/i09/>

### See Also

[kern.den.circ](#), [bw.rt](#), [bw.CV](#), [bw.boot](#)

### Examples

```
set.seed(2012)
n <- 100
x <- rcircmix(n,model=18)
bw.pi(x, M=3)
bw.pi(x, outM=TRUE) # Using AIC
```

---

bw.reg.circ.lin

*Cross-validation rule for circular regression estimation*

---

### Description

Function `bw.reg.circ.lin` provides the least squares cross-validation smoothing parameter for the Nadaraya-Watson and Local-Linear estimators when the covariate is circular and the response variable is linear.

Function `bw.reg.circ.circ` provides the least squares cross-validation smoothing parameter for the Nadaraya-Watson and Local-Linear estimators when the covariate and the response variable are circular.

Function `bw.reg.lin.circ` provides the least squares cross-validation smoothing parameter for the Nadaraya-Watson and Local-Linear estimators when the covariate is linear and the response variable is circular.

### Usage

```
bw.reg.circ.lin(x, y, method="LL", lower=0, upper=50, tol=1e-2)
bw.reg.circ.circ(x, y, method="LL", option=1, lower=0, upper=50, tol=1e-2)
bw.reg.lin.circ(x, y, method="LL", option=1, lower=0, upper=50, tol=1e-2)
```

### Arguments

<code>x</code>	Vector of data for the independent variable. The object is coerced to class <code>circular</code> when using functions <code>bw.reg.circ.lin</code> and <code>bw.reg.circ.circ</code> .
<code>y</code>	Vector of data for the dependent variable. This must be same length as <code>x</code> . The object is coerced to class <code>circular</code> when using functions <code>bw.reg.circ.circ</code> and <code>bw.reg.lin.circ</code> .
<code>method</code>	Character string giving the estimator to be used. This must be one of "LL" or "NW". Default <code>method="LL"</code> .
<code>option</code>	Cross-validation rule. Default <code>option=1</code> . See details.
<code>lower, upper</code>	lower and upper boundary of the interval to be used in the search for the value of the smoothing parameter. Default <code>lower=0</code> and <code>upper=50</code> .
<code>tol</code>	Convergence tolerance for <code>optimize</code> . Default <code>tol=1e-2</code> .

### Details

For nonparametric regression with circular response, given  $(X_i, Y_i)$ ,  $i = 1, \dots, n$ : If `option=1`, the cross-validation smoothing parameter is computed as the value that minimizes  $\sum_{i=1}^n (-\cos(Y_i - \hat{f}^{-i}(X_i)))$ , where  $\hat{f}^{-i}$  denotes the estimator computed with all the observations except  $(X_i, Y_i)$ .

If `option=2`, the cross-validation smoothing parameter is computed as the value that minimizes  $n^{-1} \sum_{i=1}^n (d(Y_i, \hat{f}^{-i}(X_i)))^2$  where  $d(Y_i, \hat{f}^{-i}(X_i)) = \min(|Y_i - \hat{f}^{-i}(X_i)|, 2\pi - |Y_i - \hat{f}^{-i}(X_i)|)$ .

The NAs will be automatically removed.

### Value

Value of the smoothing parameter.

### Author(s)

Mar?a Oliveira, Rosa M. Crujeiras and Alberto Rodr?guez-Casal

### References

Oliveira, M., Crujeiras R.M. and Rodr?guez-Casal, A. (2013) Nonparametric circular methods for exploring environmental data. *Environmental and Ecological Statistics*, **20**, 1–17.

Di Marzio, M., Panzera A. and Taylor, C. C. (2012) Non-parametric regression for circular responses. *Scandinavian Journal of Statistics*, **40**, 228–255.

Oliveira, M., Crujeiras R.M. and Rodríguez-Casal, A. (2014) NPCirc: an R package for nonparametric circular methods. *Journal of Statistical Software*, **61**(9), 1–26. <https://www.jstatsoft.org/v61/i09/>

### See Also

[kern.reg.circ.lin](#), [kern.reg.circ.circ](#), [kern.reg.lin.circ](#)

### Examples

```
set.seed(2012)
n <- 100
x <- seq(0, 2*pi, length=n)
y <- sin(x) + 0.2*rnorm(n)
bw.reg.circ.lin(circular(x), y, method="LL", lower=1, upper=20)
bw.reg.circ.lin(circular(x), y, method="NW", lower=1, upper=20)
```

---

bw.rt

*Rule of thumb*

---

### Description

This function implements the selector proposed by Taylor (2008) for density estimation, based on an estimation of the concentration parameter of a von Mises distribution. The concentration parameter can be estimated by maximum likelihood or by a robustified procedure as described in Oliveira et al. (2013).

### Usage

```
bw.rt(x, robust=FALSE, alpha=0.5)
```

### Arguments

x	Data from which the smoothing parameter is to be computed. The object is coerced to class <code>circular</code> .
robust	Logical, if <code>robust=FALSE</code> the parameter $\kappa$ is estimated by maximum likelihood, if <code>TRUE</code> it is estimated as described in Oliveira et al. (2012b). Default <code>robust=FALSE</code> .
alpha	Arc probability when <code>robust=TRUE</code> . Default is <code>alpha=0.5</code> . See Details.

### Details

When `robust=TRUE`, the parameter  $\kappa$  is estimated as follows:

1. Select  $\alpha \in (0, 1)$  and find the shortest arc containing  $\alpha \cdot 100\%$  of the sample data.
2. Obtain the estimated  $\hat{\kappa}$  in such way that the probability of a von Mises centered in the midpoint of the arc is `alpha`.

The NAs will be automatically removed.

See also Oliveira et al. (2012).

**Value**

Value of the smoothing parameter.

**Author(s)**

Mar?a Oliveira, Rosa M. Crujeiras and Alberto Rodr?guez-Casal

**References**

Oliveira, M., Crujeiras, R.M. and Rodr?guez-Casal, A. (2012) A plug-in rule for bandwidth selection in circular density. *Computational Statistics and Data Analysis*, **56**, 3898–3908.

Oliveira, M., Crujeiras R.M. and Rodr?guez-Casal, A. (2013) Nonparametric circular methods for exploring environmental data. *Environmental and Ecological Statistics*, **20**, 1–17.

Taylor, C.C. (2008) Automatic bandwidth selection for circular density estimation. *Computational Statistics and Data Analysis*, **52**, 3493–3500.

Oliveira, M., Crujeiras R.M. and Rodr?guez-Casal, A. (2014) NPCirc: an R package for nonparametric circular methods. *Journal of Statistical Software*, **61**(9), 1–26. <https://www.jstatsoft.org/v61/i09/>

**See Also**

[kern.den.circ](#), [bw.CV](#), [bw.pi](#), [bw.boot](#)

**Examples**

```
set.seed(2012)
n <- 100
x <- rcircmix(n,model=7)
bw.rt(x)
bw.rt(x, robust=TRUE)
```

---

circsizer.density      *CircSiZer map for density*

---

**Description**

This function plots the CircSiZer map for circular density estimation based on circular kernel methods, as described in Oliveira et al. (2013). The CircSiZer is an extension of SiZer proposed by Chaudhuri and Marron (1999) to circular data.

**Usage**

```
circsizer.density(x, bws, ngrid=250, alpha=0.05, B=500, log.scale=TRUE,
display=TRUE)
## S3 method for class 'circsizer'
print(x, digits=NULL, ...)
```

**Arguments**

<code>x</code>	Data from which the estimate is to be computed. The object is coerced to class <code>circular</code> .
<code>bws</code>	Vector of smoothing parameters. Values of <code>bws</code> must be positive. <code>bws</code> will be coerced to be equally spaced. Length of vector <code>bws</code> must be at least 2.
<code>ngrid</code>	Integer indicating the number of equally spaced angles between 0 and $2\pi$ where the estimator is evaluated. Default <code>ngrid=250</code> .
<code>alpha</code>	Significance level for the CircSiZer map. Default <code>alpha=0.05</code> .
<code>B</code>	Integer indicating the number of bootstrap samples to estimate the standard deviation of the derivative estimator. Default <code>B=500</code> .
<code>log.scale</code>	Logical, if TRUE, the CircSiZer map is plotted in the scale $-\log_{10}(bws)$ . Default is TRUE. See Details.
<code>display</code>	Logical, if TRUE, the CircSiZer map is plotted. Default is TRUE.
<code>digits</code>	Integer indicating the precision to be used.
<code>...</code>	further arguments

**Details**

With CircSiZer, significance features (peaks and valleys) in the data are sought via the construction of confidence intervals for the scale-space version of the smoothed derivative curve, as it is described in Oliveira et al. (2013). Thus, for a given point and a given value of the smoothing parameter, the curve is significantly increasing (decreasing) if the confidence interval is above (below) 0 and if the confidence interval contains 0, the curve for that value of the smoothing parameter and at that point does not have a statistically significant slope. If `display=TRUE`, this information is displayed in a circular color map, the CircSiZer map, in such a way that, at a given point, the performance of the estimated curve is represented by a color ring with radius proportional to the value of the smoothing parameter.

Different colors allow to identify peaks and valleys. Blue color indicates locations where the curve is significantly increasing; red color shows where it is significantly decreasing and purple indicates where it is not significantly different from zero. Gray color corresponds to those regions where there is not enough data to make statements about significance. Thus, at a given bandwidth, a significant peak can be identified when a region of significant positive gradient is followed by a region of significant negative gradient (i.e. blue-red pattern), and a significant trough by the reverse (red-blue pattern), taking clockwise as the positive sense of rotation.

If `log.scale=TRUE` then, the values of the considered smoothing parameters `bws` are transformed to  $-\log_{10}$  scale, i.e, a sequence of equally spaced smoothing parameters according to the parameters  $-\log_{10}(\max(bws))$ ,  $-\log_{10}(\min(bws))$  and `length(bws)` is used. Hence, small values of this parameter corresponds with larger rings and large values corresponds with smaller rings. Whereas if `log.scale=FALSE`, small values of this parameter corresponds with smaller rings and large values corresponds with larger rings.

The NAs will be automatically removed.

**Value**

An object with class `circsizer` whose underlying structure is a list containing the following components:

data	Original dataset.
ngrid	Number of equally spaced angles where the derivative of the circular kernel density estimator.
bw	Vector of smoothing parameters (given in $-\log_{10}$ scale if <code>log.scale=TRUE</code> ).
log.scale	Logical; if TRUE, the $-\log_{10}$ scale is used for constructing the CircSiZer map.
CI	List containing: a matrix with lower limits for the confidence intervals; a matrix with the lower limits of the confidence intervals; a matrix with the Effective Sample Size. Each row corresponds to each value of the smoothing parameter and each column corresponds to an angle.
col	Matrix containing the colors for plotting the CircSiZer map.

If `display==TRUE`, the function also returns the CircSiZer map for density.

### Author(s)

Mar?a Oliveira, Rosa M. Crujeiras and Alberto Rodr?guez-Casal

### References

Chaudhuri, P. and Marron, J.S. (1999). SiZer for exploration of structures in curves, *Journal of the American Statistical Association*, **94**, 807–823.

Oliveira, M., Crujeiras, R.M. and Rodr?guez-Casal (2014) CircSiZer: an exploratory tool for circular data. *Environmental and Ecological Statistics*, **21**, 143–159.

Oliveira, M., Crujeiras R.M. and Rodr?guez-Casal, A. (2014) NPCirc: an R package for nonparametric circular methods. *Journal of Statistical Software*, **61**(9), 1–26. <https://www.jstatsoft.org/v61/i09/>

### See Also

[circsizer.map](#)

### Examples

```
## Not run:
set.seed(2012)
x <- rcircmix(100,model=7)
sizer <- circsizer.density(x, bws=seq(0,50,length=12))
sizer
names(sizer)
circsizer.map(sizer,type=1,zero=pi/2,clockwise=TRUE,raw.data=TRUE)

## End(Not run)
```

---

circsizer.map                      *CircSiZer map*

---

### Description

This function plots the CircSiZer map for circsizer objects.

### Usage

```
circsizer.map(circsizer.object, type, zero, clockwise, title=NULL, labels=NULL,
label.pos=NULL, rad.pos=NULL, raw.data=FALSE)
```

### Arguments

circsizer.object	An object of class circsizer, i.e., output from functions circsizer.density or circsizer.regression.
type	Number indicating the labels to display in the plot: 1 (directions), 2 (hours), 3 (angles in radians), 4 (angles in degrees) or 5 (months).
zero	Where to place the starting (zero) point.
clockwise	Whether to interpret positive positions as clockwise from the starting point.
title	Title for the plot.
labels	Character or expression vector of labels to be placed at the label.pos. label.pos must also be supplied.
label.pos	Vector indicating the position (between 0 and $2\pi$ ) at which the labels are to be drawn.
rad.pos	Vector (between 0 and $2\pi$ ) with the drawing position for the radius.
raw.data	Logical, if TRUE, points indicated by x are stacked on the perimeter of the circle. Default is FALSE.

### Value

CircSiZer map.

### Author(s)

Mar?a Oliveira, Rosa M. Crujeiras and Alberto Rodr?guez-Casal

### References

Oliveira, M., Crujeiras R.M. and Rodr?guez-Casal, A. (2014) NPCirc: an R package for nonparametric circular methods. *Journal of Statistical Software*, **61**(9), 1–26. <https://www.jstatsoft.org/v61/i09/>

### See Also

[circsizer.density](#), [circsizer.regression](#)



---

circsizer.regression *CircSiZer map for regression*

---

### Description

This function plots the CircSiZer map for circular regression estimation based on circular kernel methods, as described in Oliveira et al. (2013). The CircSiZer is an extension of SiZer proposed by Chaudhuri and Marron (1999) to circular data.

### Usage

```
circsizer.regression(x, y, bws=NULL, adjust=2, ngrid=150, alpha=0.05, B=500,
B2=250, log.scale=TRUE, display=TRUE)
```

### Arguments

x	Vector of data for the independent variable. The object is coerced to class <a href="#">circular</a> .
y	Vector of data for the dependent variable. This must be same length as x.
bws	Vector of smoothing parameters. Values of bws must be positive. bws will be coerced to be equally spaced. Length of vector bws must be at least 2.
adjust	If bws=NULL, the smoothing parameters used are adjust/bw and adjust*bw, where bw is the smoothing parameter obtained by using the cross-validation rule.
ngrid	Integer indicating the number of equally spaced angles between 0 and $2\pi$ where the estimator is evaluated. Default ngrid=150.
alpha	Significance level for the CircSiZer map. Default alpha=0.05.
B	Integer indicating the number of bootstrap samples to estimate the standard deviation of the derivative estimator. Default B=500.
B2	Integer indicating the number of bootstrap samples to compute the denominator in Step 2 of algorithm described in Oliveira et al. (2013). Default B=250.
log.scale	Logical, if TRUE, the CircSiZer map is plotted in the scale $-\log_{10}(bws)$ . Default is TRUE.
display	Logical, if TRUE, the CircSiZer map is plotted. Default is TRUE.

### Details

See Details Section of [circsizer.density](#). The NAs will be automatically removed.

### Value

An object with class `circsizer` whose underlying structure is a list containing the following components.

data	Original dataset.
------	-------------------

ngrid	Number of equally spaced angles where the derivative of the regression estimator is evaluated.
bw	Vector of smoothing parameters (given in $-\log_{10}$ scale if <code>log.scale=TRUE</code> ).
log.scale	Logical; if TRUE, the $-\log_{10}$ scale is used for constructing the CircSiZer map.
CI	List containing: a matrix with lower limits for the confidence intervals; a matrix with the lower limits of the confidence intervals; a matrix with the Effective Sample Size. Each row corresponds to each value of the smoothing parameter and each column corresponds to an angle.
col	Matrix containing the colors for plotting the CircSiZer map.

If `display==TRUE`, the function also returns the CircSiZer map for regression.

### Author(s)

Mar?a Oliveira, Rosa M. Crujeiras and Alberto Rodr?guez-Casal

### References

Chaudhuri, P. and Marron, J.S. (1999). SiZer for exploration of structures in curves, *Journal of the American Statistical Association*, **94**, 807–823.

Oliveira, M., Crujeiras, R.M. and Rodr?guez-Casal (2014) CircSiZer: an exploratory tool for circular data. *Environmental and Ecological Statistics*, **21**, 143–159.

Oliveira, M., Crujeiras R.M. and Rodr?guez-Casal, A. (2014) NPCirc: an R package for nonparametric circular methods. *Journal of Statistical Software*, **61**(9), 1–26. <https://www.jstatsoft.org/v61/i09/>

### See Also

[circsizer.map](#)

### Examples

```
## Not run:
set.seed(2012)
n <- 100
x <- seq(0, 2*pi, length=n)
y <- sin(x)+sqrt(0.5)*rnorm(n)
circsizer.regression(circular(x), y, bws=seq(10,60,by=5))

## End(Not run)
```

---

cross.beds1	<i>Cross-beds azimuths (I)</i>
-------------	--------------------------------

---

**Description**

This dataset corresponds to azimuths of cross-beds in the Kamthi river (India). Originally analyzed by SenGupta and Rao (1966) and included in Table 1.5 in Mardia (1972), the dataset collects 580 azimuths of layers lying oblique to principal accumulation surface along the river, being these layers known as cross-beds.

**Usage**

```
data(cross.beds1)
```

**Format**

A single-column data frame with 580 observations in radians.

**Details**

Data were originally recorded in degrees.

**Source**

Mardia, K.V. (1972) *Statistics of Directional Data*. Academic Press, New York.

SenGupta, S. and Rao, J.S. (1966) Statistical analysis of cross-bedding azimuths from the Kamthi formation around Bheemaram, Pranhita: Godavari Valley. *Sankhya: The Indian Journal of Statistics, Series B*, **28**, 165–174.

**Examples**

```
data(cross.beds1)
```

---

cross.beds2	<i>Cross-beds (II)</i>
-------------	------------------------

---

**Description**

A dataset of cross-beds measurements from Himalayan molasse in Pakistan presented in Fisher (1993). This dataset collects 104 measurements of Chaudan Zam large bedforms.

**Usage**

```
data(cross.beds2)
```

**Format**

A single-column data frame with 104 observations in radians.

**Details**

Data were originally recorded in degrees.

**Source**

Fisher, N.I. (1993) *Statistical Analysis of Circular Data*. Cambridge University Press, Cambridge, U.K.

**Examples**

```
data(cross.beds2)
```

---

cycle.changes

*Cycle changes*

---

**Description**

The data consists on the changes in cycles of temperatures at ground level in periglacial Monte Alvear (Argentina). The dataset includes 350 observations which correspond to the hours where the temperature changes from positive to negative and viceversa from February 2008 to December 2009.

**Usage**

```
data(cycle.changes)
```

**Format**

A data frame with 350 observations on two variables: change, which indicates if the temperature changed from positive to negative (-1) or viceversa (1) and hour, which indicates the hour (in radians) when the cycle change occurred.

**Details**

Analysis of cycle changes in temperatures for another locations can be seen in Oliveira et al. (2013).

**Source**

The authors want to acknowledge Prof. Augusto Pérez-Alberti for providing the data, collected within the Project POL2006-09071 from the Spanish Ministry of Education and Science.

**References**

Oliveira, M., Crujeiras R.M. and Rodríguez-Casal, A. (2013) Nonparametric circular methods for exploring environmental data. *Environmental and Ecological Statistics*, **20**, 1–17.

## Examples

```
data(cycle.changes)
thaw <- (cycle.changes[,1]==1)
frosting <- (cycle.changes[,1]==-1)
plot(circular(cycle.changes[frosting,2],template="clock24"), shrink=1.08, col=4,
stack=TRUE, main="Frosting")
plot(circular(cycle.changes[thaw,2],template="clock24"), shrink=1.08, col=2,
stack=TRUE, main="Thaw")
```

---

dcircmix

*Mixtures of circular distributions*


---

## Description

Density and random generation functions for a circular distribution or a mixture of circular distributions allowing the following components: circular uniform, von Mises, cardioid, wrapped Cauchy, wrapped normal, wrapped skew-normal.

## Usage

```
dcircmix(x, model=NULL, dist=NULL, param=NULL)
rcircmix(n, model=NULL, dist=NULL, param=NULL)
```

## Arguments

x	Vector of angles where the density is evaluated. The object is coerced to class <a href="#">circular</a> .
n	Number of observations to generate.
model	Number between 1 and 20, corresponding with a model defined in Oliveira et al. (2012). See Details.
dist	Vector of strings with the distributions that participate in the mixture: "unif", "vm", "car", "wc", "wn", "wsn".
param	List with three or four objects. The first object will be a vector containing the proportion of each distribution in the mixture, the second object will be a vector containing the location parameters and the third object will be a vector containing the concentration parameters. If the wrapped skew-normal distribution participates in the mixture, a fourth object will be introduced in the list, a vector containing the skewness parameter. In this case, the values of the skewness parameter for the rest of distributions in the mixture will be zero. The length of each object in the list must be equal to the length of argument dist. See Details and Examples.

## Details

Models from Oliveira et al. (2012) are described below:

M1: Circular uniform.

M2: von Mises:  $vM(\pi, 1)$ .

M3: Wrapped normal:  $WN(\pi, 0.9)$ .

M4: cardioid:  $C(\pi, 0.5)$ .

M5: Wrapped Cauchy:  $WC(\pi, 0.8)$ .

M6: Wrapped skew-normal:  $WSN(\pi, 1, 20)$ .

M7: Mixture of two von Mises  $1/2vM(0, 4) + 1/2vM(\pi, 4)$ .

M8: Mixture of two von Mises  $1/2vM(2, 5) + 1/2vM(4, 5)$ .

M9: Mixture of two von Mises  $1/4vM(0, 2) + 3/4vM(\pi/\sqrt{3}, 2)$ .

M10: Mixture of von Mises and wrapped Cauchy  $4/5vM(\pi, 5) + 1/5WC(4\pi/3, 0.9)$ .

M11: Mixture of three von Mises  $1/3vM(\pi/3, 6) + 1/3vM(\pi, 6) + 1/3vM(5\pi/3, 6)$ .

M12: Mixture of three von Mises  $2/5vM(\pi/2, 4) + 1/5vM(\pi, 5) + 2/5vM(3\pi/2, 4)$ .

M13: Mixture of three von Mises  $2/5vM(0.5, 6) + 2/5vM(3, 6) + 1/5vM(5, 24)$ .

M14: Mixture of four von Mises  $1/4vM(0, 12) + 1/4vM(\pi/2, 12) + 1/4vM(\pi, 12) + 1/4vM(3\pi/2, 12)$ .

M15: Mixture of wrapped Cauchy, wrapped normal, von Mises and wrapped skew-normal  $3/10WC(\pi - 1, 0.6) + 1/4WN(\pi + 0.5, 0.9) + 1/4vM(\pi + 2, 3) + 1/5WSN(6, 1, 3)$ .

M16: Mixture of five von Mises  $1/5vM(\pi/5, 18) + 1/5vM(3\pi/5, 18) + 1/5vM(\pi, 18) + 1/5vM(7\pi/5, 18) + 1/5vM(9\pi/5, 18)$ .

M17: Mixture of cardioid and wrapped Cauchy  $2/3C(\pi, 0.5) + 1/3WC(\pi, 0.9)$ .

M18: Mixture of four von Mises  $1/2vM(\pi, 1) + 1/6vM(\pi - 0.8, 30) + 1/6vM(\pi, 30) + 1/vM(\pi + 0.8, 30)$ .

M19: Mixture of five von Mises  $4/9vM(2, 3) + 5/36vM(4, 3) + 5/36vM(3.5, 50) + 5/36vM(4, 50) + 5/36vM(4.5, 50)$ .

M20: Mixture of two wrapped skew-normal and two wrapped Cauchy  $1/3WSN(0, 0.7, 20) + 1/3WSN(\pi, 0.7, 20) + 1/6WC(3\pi/4, 0.9) + 1/6WC(7\pi/4, 0.9)$ .

When the wrapped skew-normal distribution participates in the mixture, the argument param for function `dcircmix` can be a list with fifth objects. The fifth object would be the number of terms to be used in approximating the density function of the wrapped skew normal distribution. By default the number of terms used is 20.

## Value

`dcircmix` gives the density and `rcircmix` generates random deviates.

## Author(s)

Mar?a Oliveira, Rosa M. Crujeiras and Alberto Rodr?guez-Casal

## References

Oliveira, M., Crujeiras, R.M. and Rodríguez-Casal, A. (2012) A plug-in rule for bandwidth selection in circular density. *Computational Statistics and Data Analysis*, **56**, 3898–3908.

Oliveira, M., Crujeiras R.M. and Rodríguez-Casal, A. (2014) NPCirc: an R package for nonparametric circular methods. *Journal of Statistical Software*, **61**(9), 1–26. <https://www.jstatsoft.org/v61/i09/>

## Examples

```
set.seed(2012)
# Circular representation of models M1-M20, each one in a separate window
for (i in 1:20){
  dev.new()
  f <- function(x) dcircmix(x, model=i)
  curve.circular(f, n=500, join=TRUE, shrink=1.9, main=i)
}

# Random generation from model M1 (uniform model)
data1 <- rcircmix(50, model=1)
plot(data1)

# Density function and random generation from a mixture of a von Mises and
# a wrapped skew-normal
f <- function(x) dcircmix(x, model=NULL, dist=c("vm", "wsn"),
  param=list(p=c(0.5,0.5), mu=c(0,pi), con=c(1,1), sk=c(0,10)))
curve.circular(f, n=500, shrink=1.2)
data <- rcircmix(100, model=NULL, dist=c("vm", "wsn"),
  param=list(p=c(0.5,0.5), mu=c(0,pi), con=c(1,1), sk=c(0,10)))
points(data)

# Density function and random generation from a mixture of two von Mises and
# two wrapped Cauchy
f <- function(x) dcircmix(x, model=NULL, dist=c("vm", "vm", "wc", "wc"),
  param=list(p=c(0.3,0.3,0.2,0.2), mu=c(0,pi,pi/2,3*pi/2), con=c(5,5,0.9,0.9)))
curve.circular(f, n=1000, xlim=c(-1.65,1.65))
data <- rcircmix(100, model=NULL, dist=c("vm", "vm", "wc", "wc"),
  param=list(p=c(0.3,0.3,0.2,0.2), mu=c(0,pi,pi/2,3*pi/2), con=c(5,5,0.9,0.9)))
points(data)
```

---

dragonfly

*Orientations of dragonflies*

---

## Description

The data, presented in Batschelet (1981), consists on the orientation of 214 dragonflies with respect to the sun's azimuth.

## Usage

```
data(dragonfly)
```

**Format**

A single-column data frame with 214 observations in radians.

**Details**

Data were originally recorded in degrees.

**Source**

Batschelet, E. (1981) *Circular Statistics in Biology*. Academic Press, New York.

**Examples**

```
data(dragonfly)
x <- circular(dragonfly$orientation)
dens <- kern.den.circ(x)
plot(dens, shrink=1.3)
```

---

dwsn

*Wrapped skew-Normal density function*


---

**Description**

Density function and random generation for the wrapped skew-Normal distribution introduced by Pewsey (2000).

**Usage**

```
dwsn(x, xi, eta, lambda, K=NULL, min.k=20)
rwsn(n, xi, eta, lambda)
```

**Arguments**

x	Vector of angles where the density is evaluated . The object is coerced to class <a href="#">circular</a> .
n	Number of observations.
xi	Location parameter. The object is coerced to class <a href="#">circular</a> .
eta	Scale parameter.
lambda	Skewness parameter.
K	Number of terms to be used in approximating the density. Default K=NULL.
min.k	Minimum number of terms used in approximating the density.

**Details**

The NAs will be automatically removed.



**Value**

dwsn gives the density and rwsn generates random deviates.

**Author(s)**

María Oliveira, Rosa M. Crujeiras and Alberto Rodríguez-Casal

**References**

Pewsey, A. (2000) The wrapped skew-Normal distribution on the circle. *Communications in Statistics - Theory and Methods*, **29**, 2459–2472.

**Examples**

```
set.seed(2012)
# Density function of a wrapped skew-normal distribution WSN(pi,1,20)
wsn <- function(x) dwsn(x, xi=circular(pi), eta=1, lambda=20)
curve.circular(wsn,n=500,xlim=c(-1.65,1.65),main=expression(WSN(pi,1,20)))
# Random generation
data<-rwsn(50,xi=circular(pi),eta=1,lambda=20)
points(data)
```

---

kern.den.circ

*Nonparametric circular kernel density estimation*


---

**Description**

This function computes circular kernel density estimates with the given bandwidth, taking the von Mises distribution as circular kernel.

**Usage**

```
kern.den.circ(x, t=NULL, bw=NULL, from=circular(0), to=circular(2*pi), len=250)
```

**Arguments**

x	Data from which the estimate is to be computed. The object is coerced to class <code>circular</code> .
t	Points where the density is estimated. If NULL equally spaced points are used according to the parameters from, to and len.
bw	Smoothing parameter to be used. The value of the smoothing parameter can be chosen by using the functions <code>bw.rt</code> , <code>bw.CV</code> , <code>bw.pi</code> and <code>bw.boot</code> .
from, to	Left and right-most points of the grid at which the density is to be estimated. The objects are coerced to class <code>circular</code> .
len	Number of equally spaced points at which the density is to be estimated.

**Details**

The NAs will be automatically removed.

**Value**

An object with class `density.circular` whose underlying structure is a list containing the following components:

<code>data</code>	Original dataset.
<code>x</code>	The points where the density is estimated.
<code>y</code>	The estimated density values.
<code>bw</code>	The smoothing parameter used.
<code>N</code>	The sample size after elimination of missing values.
<code>call</code>	The call which produced the result.
<code>data.name</code>	The deparsed name of the x argument.
<code>has.na</code>	Logical, for compatibility (always FALSE).

**Author(s)**

Mar?a Oliveira, Rosa M. Crujeiras and Alberto Rodr?guez-Casal

**References**

- Oliveira, M., Crujeiras, R.M. and Rodr?guez-Casal, A. (2012) A plug-in rule for bandwidth selection in circular density. *Computational Statistics and Data Analysis*, **56**, 3898–3908.
- Taylor, C.C. (2008) Automatic bandwidth selection for circular density estimation. *Computational Statistics and Data Analysis*, **52**, 3493–3500.
- Oliveira, M., Crujeiras R.M. and Rodr?guez-Casal, A. (2014) NPCirc: an R package for nonparametric circular methods. *Journal of Statistical Software*, **61**(9), 1–26. <https://www.jstatsoft.org/v61/i09/>

**See Also**

[bw.rt](#), [bw.CV](#), [bw.pi](#), [bw.boot](#), [plot.density.circular](#), [lines.density.circular](#)

**Examples**

```
set.seed(2012)
n <- 100
x <- rcircmix(n, model=14)
est1 <- kern.den.circ(x, t=NULL, bw=NULL)
plot(est1, plot.type="circle", points.plot=TRUE, shrink=1.2, main="Circular plot")
est2 <- kern.den.circ(x, t=NULL, bw=20)
lines(est2, plot.type="circle", shrink=1.2, col=2)
plot(est1, plot.type="line", main="Linear plot")
lines(est2, plot.type="line", col=2)
```

---

kern.reg.circ.lin      *Nonparametric regression estimation for circular data*

---

## Description

Function `kern.reg.circ.lin` implements the Nadaraya-Watson estimator and the Local-Linear estimator for circular-linear data (circular covariate and linear response), as described in Di Marzio et al. (2009) and Oliveira et al. (2013), taking the von Mises distribution as kernel.

Function `kern.reg.circ.circ` implements the Nadaraya-Watson estimator and the Local-Linear estimator for circular-circular data (circular covariate and circular response), as described in Di Marzio et al. (2012), taking the von Mises distribution as kernel.

Function `kern.reg.lin.circ` implements the Nadaraya-Watson estimator and the Local-Linear estimator for linear-circular data (linear covariate and circular response), as described in Di Marzio et al. (2012), taking the Normal distribution as kernel.

## Usage

```
kern.reg.circ.lin(x, y, t=NULL, bw, method="LL", from=circular(0),
to=circular(2*pi), len=250, tol=300)
kern.reg.circ.circ(x, y, t=NULL, bw, method="LL", from=circular(0),
to=circular(2*pi), len=250)
kern.reg.lin.circ(x, y, t=NULL, bw, method="LL", len=250)
## S3 method for class 'regression.circular'
print(x, digits=NULL, ...)
```

## Arguments

<code>x</code>	Vector of data for the independent variable. The object is coerced to class <code>circular</code> when using functions <code>kern.reg.circ.lin</code> and <code>kern.reg.circ.circ</code> .
<code>y</code>	Vector of data for the dependent variable. This must be same length as <code>x</code> . The object is coerced to class <code>circular</code> when using functions <code>kern.reg.circ.circ</code> and <code>kern.reg.lin.circ</code> .
<code>t</code>	Points where the regression function is estimated. If <code>NULL</code> equally spaced points are used according to the parameters <code>from</code> , <code>to</code> and <code>len</code> .
<code>bw</code>	Smoothing parameter to be used. The value of the smoothing parameter can be chosen by using the function <code>bw.reg.circ.lin</code> , <code>bw.reg.circ.circ</code> and <code>bw.reg.lin.circ</code> .
<code>method</code>	Character string giving the estimator to be used. This must be one of "LL" for Local-Linear estimator or "NW" for Nadaraya-Watson estimator. Default <code>method="LL"</code> .
<code>from, to</code>	Left and right-most points of the grid at which the density is to be estimated. The objects are coerced to class <code>circular</code> .
<code>len</code>	Number of equally spaced points at which the density is to be estimated.
<code>tol</code>	Tolerance parameter to avoid overflow when <code>bw</code> is larger than <code>tol</code> . Default is <code>tol=300</code> .

digits	Integer indicating the precision to be used.
...	further arguments

### Details

See Di Marzio et al. (2012). See Section 3 in Oliveira et al. (2013). See Di Marzio et al. (2009). The NAs will be automatically removed.

### Value

An object with class "regression.circular" whose underlying structure is a list containing the following components:

data	Original dataset.
x	The n coordinates of the points where the regression is estimated.
y	The estimated values.
bw	The smoothing parameter used.
N	The sample size after elimination of missing values.
call	The call which produced the result.
data.name	The deparsed name of the x argument.
has.na	Logical, for compatibility (always FALSE).

### Author(s)

Mar?a Oliveira, Rosa M. Crujeiras and Alberto Rodr?guez-Casal

### References

- Di Marzio, M., Panzera A. and Taylor, C. C. (2009) Local polynomial regression for circular predictors. *Statistics and Probability Letters*, **79**, 2066–2075.
- Di Marzio, M., Panzera A. and Taylor, C. C. (2012) Non-parametric regression for circular responses. *Scandinavian Journal of Statistics*, **40**, 228–255.
- Oliveira, M., Crujeiras R.M. and Rodr?guez-Casal, A. (2013) Nonparametric circular methods for exploring environmental data. *Environmental and Ecological Statistics*, **20**, 1–17.
- Oliveira, M., Crujeiras R.M. and Rodr?guez-Casal, A. (2014) NPCirc: an R package for nonparametric circular methods. *Journal of Statistical Software*, **61**(9), 1–26. <https://www.jstatsoft.org/v61/i09/>

### See Also

[plot.regression.circular](#), [lines.regression.circular](#)

**Examples**

```

### circular-linear
data(speed.wind2)
dir <- speed.wind2$Direction
vel <- speed.wind2$Speed
nas <- which(is.na(vel))
dir <- circular(dir[-nas],units="degrees")
vel <- vel[-nas]
estLL <- kern.reg.circ.lin(dir, vel, method="LL")
estNW <- kern.reg.circ.lin(dir, vel, method="NW")
# Circular representation
res<-plot(estNW, plot.type="circle", points.plot=TRUE,
labels=c("N", "NE", "E", "SE", "S", "SO", "O", "NO"),
label.pos=seq(0,7*pi/4,by=pi/4), zero=pi/2, clockwise=TRUE)
lines(estLL, plot.type="circle", plot.info=res, line.col=2)
# Linear representation
plot(estNW, plot.type="line", points.plot=TRUE, xlab="direction", ylab="speed (m/s)")
lines(estLL, plot.type="line", line.col=2)

### circular-circular
data(wind)
wind6 <- circular(wind$wind.dir[seq(7,1752,by=24)])
wind12 <- circular(wind$wind.dir[seq(13,1752,by=24)])
estNW <- kern.reg.circ.circ(wind6,wind12,t=NULL,bw=6.1,method="NW")
estLL <- kern.reg.circ.circ(wind6,wind12,t=NULL,bw=2.25,method="LL")
# Torus representation
plot(estNW, plot.type="circle", points.plot=TRUE, line.col=2, lwd=2, points.col=2,
units="degrees")
lines(estLL, plot.type="circle", line.col=3, lwd=2)
# Linear representation
plot(estNW, plot.type="line", points.plot=TRUE, xlab="Wind direction at 6 a.m.",
ylab="Wind direction at noon")
lines(estLL, plot.type="line", line.col=2)

### linear-circular
data(periwinkles)
dist <- periwinkles$distance
dir <- circular(periwinkles$direction, units="degrees")
estNW <- kern.reg.lin.circ(dist,dir,t=NULL,bw=12.7,method="NW")
estLL <- kern.reg.lin.circ(dist,dir,t=NULL,bw=200,method="LL")
# Cylinder representation
plot(estNW, plot.type="circle", points.plot=TRUE, line.col=2, lwd=2, points.col=2)
lines(estLL, plot.type="circle", line.col=3, lwd=2)
# Linear representation
plot(estNW, plot.type="line", points.plot=TRUE, units="radians", main="")
lines(estLL, plot.type="line", line.col=2, units="radians")

```

---

lines.regression.circular

*Add a plot for circular regression*

---

## Description

The lines add a plot for regression.circular objects.

## Usage

```
## S3 method for class 'regression.circular'
lines(x, plot.type=c("circle", "line"), points.plot=FALSE, rp.type="p", type="l",
line.col=1, points.col="grey", points.pch=1, units=NULL, zero=NULL,
clockwise=NULL, radial.lim=NULL, plot.info=NULL, ...)
```

## Arguments

x	An object of class regression.circular.
plot.type	Type of the plot: "line": linear plot, "circle": circular plot.
points.plot	Logical; if TRUE original data are added to the plot.
rp.type, type	Character indicating the type of plotting.
line.col	Color code or name.
points.col	Color code or name for the original data. Used if points.plot=TRUE.
points.pch	Plotting 'character', i.e., symbol to use for the original data. Used if points.plot=TRUE.
units	Units measure used in the plot. If NULL the value is taken from the attribute of object 'x' from the argument 'x', i.e. x\$x.
zero	Where to place the starting (zero) point, i.e., the zero of the plot. Ignored if plot.info is provided.
clockwise	Logical, indicating the sense of rotation of the plot: clockwise if TRUE and counterclockwise if FALSE. Ignored if plot.info is provided.
radial.lim	The range of the grid circle. Used if plot.type="circle".
plot.info	An object from plot.regression.circular that contains information on the zero, the clockwise and radial.lim. Used if plot.type="circle".
...	Further arguments to be passed to lines.default (if plot.type="line") or to radial.plot (if plot.type="circle" and x is the output of kern.reg.circ.lin) or to lines3d (if plot.type="circle") and x is the output of kern.reg.circ.circ).

## Author(s)

Mar?a Oliveira, Rosa M. Crujeiras and Alberto Rodr?guez-Casal

## References

Oliveira, M., Crujeiras R.M. and Rodr?guez-Casal, A. (2014) NPCirc: an R package for nonparametric circular methods. *Journal of Statistical Software*, **61**(9), 1–26. <https://www.jstatsoft.org/v61/i09/>

**See Also**

[kern.reg.circ.lin](#), [kern.reg.circ.circ](#), [kern.reg.lin.circ](#), [plot.regression.circular](#)

**Examples**

```
set.seed(1012)
n <- 100
x <- runif(100, 0, 2*pi)
y <- sin(x)+0.5*rnorm(n)
estNW<-kern.reg.circ.lin(circular(x),y,t=NULL,bw=10,method="NW")
estLL<-kern.reg.circ.lin(circular(x),y,t=NULL,bw=10,method="LL")
res<-plot(estNW, plot.type="circle", points.plot=TRUE)
lines(estLL, plot.type="circle",line.col=2, plot.info=res)
```

---

noeffect.circ.lin

*No-effect test for regression with circular data*

---

**Description**

Function `noeffect.circ.lin` computes the no-effect test for a circular predictor variable and a real-valued response variable as described in Alonso-Pena et al. (2021). It uses the nonparametric Nadaraya-Watson estimator or the Local-Linear estimator for circular-linear data described in Di Marzio et al. (2009) and Oliveira et al. (2013).

Function `noeffect.lin.circ` computes the no-effect test for a real-valued predictor variable and a circular response variable as described in Alonso-Pena et al. (2021). It uses the nonparametric Nadaraya-Watson estimator or the Local-Linear estimator for linear-circular data described in Di Marzio et al. (2012).

Function `noeffect.circ.circ` computes the no-effect test for a circular predictor variable and a circular response variable as described in Alonso-Pena et al. (2021). It uses the nonparametric Nadaraya-Watson estimator or the Local-Linear estimator for circular-circular data described in Di Marzio et al. (2012).

**Usage**

```
noeffect.circ.lin(x, y, bw, method = "LL", calib = "chisq", n_boot = 500)
noeffect.lin.circ(x, y, bw, method = "LL", n_boot = 500)
noeffect.circ.circ(x, y, bw, method = "LL", n_boot = 500)
```

**Arguments**

`x` Vector of data for the independent variable. The object is coerced to class `circular` when using functions `noeffect.circ.lin` and `noeffect.circ.circ`.

`y` Vector of data for the dependent variable. This must be same length as `x`. The object is coerced to class `circular` when using functions `noeffect.lin.circ` and `noeffect.circ.circ`.

bw	Smoothing parameter to be used. If not provided, functions <code>noeffect.circ.lin</code> and <code>noeffect.circ.circ</code> select $4cv$ and function <code>noeffect.lin.circ</code> selects $cv/4$ , where $cv$ is the parameter selected by cross-validation.
method	Character string giving the estimator to be used. This must be one of "LL" for Local-Linear estimator or "NW" for Nadaraya-Watson estimator. Default <code>method="LL"</code> .
calib	Character string giving the calibration method to be used in <code>noeffect.circ.lin</code> function. This must be one of "chisq" for the chi-squared approximation or "boot" for the bootstrap calibration.
n_boot	Number of bootstrap resamples. Default is <code>n_boot=500</code> . In function <code>noeffect.circ.lin</code> , only if <code>calib="boot"</code> .

### Details

See Alonso-Pena et al. (2021). The NAs will be automatically removed.

### Value

A list with class "htest" containing the following components:

statistic	observed value of the statistic.
bw	Smoothing parameter used.
p.value	p-value for the test.
data.name	a character string giving the name(s) of the data.
alternative	a character string describing the alternative hypothesis.

### Author(s)

María Alonso-Pena, Jose Ameijeiras-Alonso and Rosa M. Crujeiras

### References

- Alonso-Pena, M., Ameijeiras-Alonso, J. and Crujeiras, R.M. (2021) Nonparametric tests for circular regression. *Journal of Statistical Computation and Simulation*, **91**(3), 477–500.
- Di Marzio, M., Panzera A. and Taylor, C. C. (2009) Local polynomial regression for circular predictors. *Statistics and Probability Letters*, **79**, 2066–2075.
- Di Marzio, M., Panzera A. and Taylor, C. C. (2012) Non-parametric regression for circular responses. *Scandinavian Journal of Statistics*, **40**, 228–255.
- Oliveira, M., Crujeiras R.M. and Rodriguez-Casal, A. (2013) Nonparametric circular methods for exploring environmental data. *Environmental and Ecological Statistics*, **20**, 1–17.

### See Also

[kern.reg.circ.lin](#), [kern.reg.lin.circ](#), [kern.reg.circ.circ](#)



**Examples**

```
# No-effect circ-lin
set.seed(2025)
x <- rcircularuniform(200)
y <- 2*sin(as.numeric(x)) + rnorm(200, sd=2)
noeffect.circ.lin(x, y)

# No-effect lin-circ
set.seed(2025)
x <- runif(200)
y <- pi/8 + rvonmises(200, mu = 0, kappa = 0.75)
noeffect.lin.circ(x, y)

# No-effect circ-circ
set.seed(2025)
x <- rcircularuniform(200)
y <- atan2(sin(2*x), cos(2*x)) + rvonmises(200, mu = 0, kappa = 2)
noeffect.circ.circ(x, y)
```

---

periwinkles

*Orientations of dragonflies*

---

**Description**

These data, presented in Fisher and Lee (1992), contain distance and directions of movements from small blue periwinkles after relocation.

**Usage**

```
data(periwinkles)
```

**Format**

A two-column data frame with 73 observations. Distances are measured in centimeters and directions are measured in degrees.

**Source**

Fisher, N. I. and Lee, A. J. (1992) Regression models for angular responses. *Biometrics*, **48**, 665–677.

**Examples**

```
data(periwinkles)
```

---

 plot.regression.circular

*Plot circular regression*


---

### Description

The plot method for regression.circular objects.

### Usage

```
## S3 method for class 'regression.circular'
plot(x, plot.type=c("circle", "line"), points.plot=FALSE, rp.type="p", type="l",
line.col=1, points.col="grey", points.pch=1, xlim=NULL, ylim=NULL,
radial.lim=NULL, xlab=NULL, ylab=NULL, labels=NULL, label.pos=NULL, units=NULL,
zero=NULL, clockwise=NULL, main=NULL, ...)
```

### Arguments

x	An object of class regression.circular.
plot.type	Type of the plot: "line": linear plot, "circle": circular plot.
points.plot	Logical; if TRUE original data are added to the plot.
rp.type, type	Character indicating the type of plotting. Default type="l" and rp.type="p".
line.col	Color code or name.
points.col	Color code or name for the original data. Used if points.plot=TRUE.
points.pch	Plotting 'character', i.e., symbol to use for the original data. Used if points.plot=TRUE.
xlim, ylim	The ranges to be encompassed by the x and y axes. Used if plot.type="line".
radial.lim	The range of the grid circle, used if plot.type="circle".
xlab, ylab	Titles for the x axis and y axis, respectively.
labels	Character or expression vector of labels to be placed at the label.pos. label.pos must also be supplied.
label.pos	Vector indicating the position (between 0 and $2\pi$ ) at which the labels are to be drawn.
units	Units measure used in the plot. If NULL the value is taken from the attribute of object 'x' from the argument 'x', i.e. x\$x.
zero	Where to place the starting (zero) point, i.e. the zero of the plot. If NULL the value is taken from the attribute of object 'x' from the argument 'x', i.e. x\$x
clockwise	Logical, indicating the sense of rotation of the plot: clockwise if TRUE and counterclockwise if FALSE. If NULL the value is taken from the attribute of object 'x' from the argument 'x', i.e. x\$x
main	An overall title for the plot.
...	Further arguments to be passed to <a href="#">plot.default</a> (if plot.type="line") or to <a href="#">radial.plot</a> (if codeplot.type="circle" and x is the output of <a href="#">kern.reg.circ.lin</a> ) or to <a href="#">lines3d</a> (if plot.type="circle") and x is the output of <a href="#">kern.reg.circ.circ</a> ).

**Value**

If `plot.type="circle"` and `x` is the output of `kern.reg.circ.lin`, this function returns a list with information on the plot: `zero`, `clockwise` and `radial.lim`.

**Author(s)**

María Oliveira, Rosa M. Crujeiras and Alberto Rodríguez-Casal

**References**

Oliveira, M., Crujeiras R.M. and Rodríguez-Casal, A. (2014) NPCirc: an R package for nonparametric circular methods. *Journal of Statistical Software*, **61**(9), 1–26. <https://www.jstatsoft.org/v61/i09/>

**See Also**

[kern.reg.circ.lin](#), [kern.reg.circ.circ](#), [kern.reg.lin.circ](#), [lines.regression.circular](#)

**Examples**

```
set.seed(1012)
n <- 100
x <- runif(100, 0, 2*pi)
y <- sin(x)+0.5*rnorm(n)
estNW<-kern.reg.circ.lin(circular(x),y,t=NULL,bw=10,method="NW")
plot(estNW, plot.type="line", points.plot=TRUE)
plot(estNW, plot.type="circle", points.plot=TRUE)
```

---

speed.wind

*Wind speed and wind direction data*

---

**Description**

This dataset consists of hourly observations of wind direction and wind speed in winter season (from November to February) from 2003 until 2012 in the Atlantic coast of Galicia (NW–Spain). Data are registered by a buoy located at longitude -0.210E and latitude 43.500N in the Atlantic Ocean. The dataset `speed.wind2`, analyzed in Oliveira et al. (2013), is a subset of `speed.wind` which is obtained by taking the observations with a lag period of 95 hours.

**Usage**

```
data(speed.wind)
data(speed.wind2)
```

**Format**

`speed.wind` is a data frame with 19488 observations on six variables: `day`, `month`, `year`, `hour`, `wind speed` (in m/s) and `wind direction` (in degrees). `speed.wind2` is a subset with 200 observations.

**Details**

Data contains NAs. There is no data in November 2005, December 2005, January 2006, February 2006, February 2007, February 2009 and November 2009. Months of November 2004, December 2004, January 2007, December 2009 are not fully observed.

**Source**

Data can be freely downloaded from the Spanish Portuary Authority (<http://www.puertos.es>).

**References**

Oliveira, M., Crujeiras, R.M. and Rodríguez-Casal (2014) CircSiZer: an exploratory tool for circular data. *Environmental and Ecological Statistics*, **21**, 143–159.

**Examples**

```
data(speed.wind2)

# Density
dir <- circular(speed.wind2$Direction, units="degrees", template="geographics")
plot(dir, stack=TRUE, shrink= 1.1)
rose.diag(dir, bins=16, add=TRUE)
lines(kern.den.circ(dir,bw=1), lwd=2, col=2)
lines(kern.den.circ(dir,bw=10), lwd=2, col=3)
lines(kern.den.circ(dir,bw=40), lwd=2, col=4)

# Regression
vel <- speed.wind2$Speed
nas <- which(is.na(vel))
dir <- dir[-nas]
vel <- vel[-nas]
res<-plot(kern.reg.circ.lin(dir, vel, bw=1, method="LL"), plot.type="circle",
points.plot=TRUE, line.col=2, lwd=2, main="")
lines(kern.reg.circ.lin(dir, vel, bw=10, method="LL"), plot.type="circle", plot.info=res,
line.col=3, lwd=2)
lines(kern.reg.circ.lin(dir, vel, bw=40, method="LL"), plot.type="circle", plot.info=res,
line.col=4, lwd=2)
```

temp.wind

*Temperature and wind direction data***Description**

These data, analyzed by Oliveira et al. (2013), consists of observations of temperature and wind direction during the austral summer season 2008-2009 (from November 2008 to March 2009) in Vinciguerra (Tierra del Fuego, Argentina).

**Usage**

```
data(temp.wind)
```

**Format**

A data frame with 3648 observations on four variables: Date, Time, Temperature (in degrees Celsius) and Direction (in degrees).

**Details**

Data contains NAs.

**Source**

The authors want to acknowledge Prof. Augusto Pérez-Alberti for providing the data, collected within the Project POL2006-09071 from the Spanish Ministry of Education and Science.

**References**

Oliveira, M., Crujeiras R.M. and Rodríguez-Casal, A. (2013) Nonparametric circular methods for exploring environmental data. *Environmental and Ecological Statistics*, **20**, 1–17.

**Examples**

```
data(temp.wind)
winddir <- temp.wind[,4]
temp <- temp.wind[,3]
nas <- which(is.na(winddir))
winddir <- circular(winddir[-nas], units="degrees")
temp <- temp[-nas]

est <- kern.reg.circ.lin(winddir, temp, t=NULL, bw=3.41, method="LL")
plot(est, plot.type="line", xlab="wind direction", ylab="temperature")
plot(est, plot.type="circle", points.plot=TRUE)
```

---

wind

*Wind direction data*

---

**Description**

This dataset consists of hourly observations of wind direction measured at a weather station in Texas from May 20 to July 31, 2003 inclusive.

**Usage**

```
data(wind)
```

**Format**

wind is a data frame with observations on three variables: data, hour and wind direction (in radians).

**Source**

The data, which corresponds to the weather station designated as C28\_1, are part of a larger data set taken from the Codiak data archive, available at <https://data.eol.ucar.edu/dataset/85.034>. The full data set contains hourly resolution surface meteorological data from the Texas Natural Resources Conservation Commission Air Quality Monitoring Network, from May 20 to July 31, 2003 inclusive. These data are provided by NCAR/EOL under the sponsorship of the National Science Foundation.

**References**

- Kato, S. and Jones, M. C. (2010) A family of distributions on the circle with links to, and applications arising from, Möbius transformation. *Journal of the American Statistical Association*, **105**, 249–262.
- Di Marzio, M., Panzera A. and Taylor, C. C. (2012) Non-parametric regression for circular responses. *Scandinavian Journal of Statistics*, **40**, 228–255.

**Examples**

```
data(wind)
```

# Index

- \* **~hstest**
  - ancova.circ.lin, 4
  - noeffect.circ.lin, 31
- \* **~smooth**
  - ancova.circ.lin, 4
  - noeffect.circ.lin, 31
- \* **circsizer**
  - circsizer.map, 16
- \* **circular density**
  - bw.boot, 6
  - bw.CV, 8
  - bw.pi, 9
  - bw.rt, 12
  - circsizer.density, 13
  - dcircmix, 21
  - dwsn, 24
  - kern.den.circ, 25
- \* **circular regression**
  - bw.reg.circ.lin, 10
  - circsizer.regression, 17
  - kern.reg.circ.lin, 27
  - lines.regression.circular, 30
  - plot.regression.circular, 34
- \* **datasets**
  - cross.beds1, 19
  - cross.beds2, 19
  - cycle.changes, 20
  - dragonfly, 23
  - periwinkles, 33
  - speed.wind, 35
  - temp.wind, 36
  - wind, 37
- ancova.circ.circ (ancova.circ.lin), 4
- ancova.circ.lin, 4
- ancova.lin.circ (ancova.circ.lin), 4
- bw.boot, 6, 9, 10, 13, 25, 26
- bw.CV, 7, 8, 10, 13, 25, 26
- bw.pi, 7, 9, 9, 13, 25, 26
- bw.reg.circ.circ, 27
- bw.reg.circ.circ (bw.reg.circ.lin), 10
- bw.reg.circ.lin, 10, 27
- bw.reg.lin.circ, 27
- bw.reg.lin.circ (bw.reg.circ.lin), 10
- bw.rt, 7, 9, 10, 12, 25, 26
- circsizer.density, 13, 16, 17
- circsizer.map, 15, 16, 18
- circsizer.regression, 16, 17
- circular, 4, 7–9, 11, 12, 14, 17, 21, 24, 25, 27, 31
- cross.beds1, 19
- cross.beds2, 19
- cycle.changes, 20
- dcircmix, 21
- density.circular, 26
- dragonfly, 23
- dwsn, 24
- kern.den.circ, 7, 9, 10, 13, 25
- kern.reg.circ.circ, 5, 12, 30–32, 34, 35
- kern.reg.circ.circ (kern.reg.circ.lin), 27
- kern.reg.circ.lin, 5, 12, 27, 30–32, 34, 35
- kern.reg.lin.circ, 5, 12, 31, 32, 35
- kern.reg.lin.circ (kern.reg.circ.lin), 27
- lines.default, 30
- lines.density.circular, 26
- lines.regression.circular, 28, 29, 35
- lines3d, 30, 34
- noeffect.circ.circ (noeffect.circ.lin), 31
- noeffect.circ.lin, 31
- noeffect.lin.circ (noeffect.circ.lin), 31
- NPCirc (NPCirc-package), 2

NPCirc-package, 2

optimize, 7–9, 11

periwinkles, 33

plot.default, 34

plot.density.circular, 26

plot.regression.circular, 28, 31, 34

print.circsizer (circsizer.density), 13

print.regression.circular  
(kern.reg.circ.lin), 27

radial.plot, 30, 34

rcircmix (dcircmix), 21

rwsn (dwsn), 24

speed.wind, 35

speed.wind2 (speed.wind), 35

temp.wind, 36

wind, 37